THE PHYSICAL SOCIETY OF LONDON.

PROCEEDINGS.

VOLUME XXXIV.—PART IV.

JUNE 15, 1922.

Price to Non-Fellows, **6s.** *net, post free* **6/3.**

Annual Subscription, **30/-** *post free, payable in advance*

Published Bi-Monthly from December to August.

LONDON:
FLEETWAY PRESS, LTD.,
8-9, DANE STREET, HIGH HOLBORN, W.C.1.

1922.

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XIX. *On the Electro-magnetic Screening of a Triode Oscillator.* *By* R. L. SMITH-ROSE, *B.Sc., D.I.C., A.R.C.S., A.M.I.E.E.* (*Communication from the National Physical Laboratory.*)

RECEIVED FEBRUARY 16, 1922.

ABSTRACT.

The results of the many experiments on screening described in the accompanying Paper show that the most complete method of screening a valve set is to enclose it in a box made of metal of suitable thickness for the frequency used, and to seal hermetically all joints in the box and its lid. The smallest crack or hole is sufficient to allow of the escape of a detectable amount of the high-frequency energy, and it is shown that iron is far more effective than copper of the same thickness in preventing direct penetration of radio-frequency magnetic fields through the metal.

While a practicable means of using an oscillator inside an hermetically sealed screening box is described for work of the highest precision, a much more convenient arrangement for the use of continuous-wave wireless direction-finding is described, with some results which have been obtained with an experimental sample in actual use for this work.

The experiments herein described were carried out at the National Physical Laboratory in connection with investigations undertaken for the Radio Research Board.

1.—*Introductory.*

THE problem of screening a body or piece of apparatus from a high-frequency electro-magnetic field is known to be very much more difficult than the protection from steady electric or magnetic fields. Electrostatic screening may be effectively carried out by completely surrounding the body with a metal covering (a Faraday cage), the thickness of which is immaterial. In cases where transparency of the screen is desirable to permit of inspection of the interior, the screen need not be absolutely continuous, *e.g.*, metal gauze or perforated sheet is effectively used to screen instruments from stray electric fields, or from capacity to external objects. Magnetic shielding of a body may be carried out by surrounding it with a heavy iron screen, which, however, only partially screens by deflecting the magnetic field through the space of greater permeability.

With an alternating magnetic field, however, the shielding effect of the iron is much more complete, and in this case, also, the shielding may be produced by other metals than iron, due to the demagnetisation by the eddy currents induced in the metal by the initial magnetic fields. Simple experiments with oscillation generators give the impression that when the frequency of the alternating magnetic field is very high, the difficulty of the screening is greatly increased. This is, however, a false conception and arises from the fact that the induced E.M.F. in any circuit placed in an alternating magnetic field is directly proportional to the frequency, and thus becomes very large at frequencies of the order employed in radio work. It is essentially this fact, together with the very high magnifications produced by multi-stage triode amplifiers, that leads to some rather surprising results when attempts are made to shield instruments from radio-frequency alternating magnetic fields. That the principles underlying the effective screening of a radio-frequency field have been somewhat misunderstood in the past is borne out by the fact that several experimenters write of employing a Faraday cage for this purpose, the cage often being constructed of wire gauze. The term " Faraday cage " can only strictly be

employed in connection with electrostatic screening, and its effect is altered immediately the field is fluctuating or alternating.

2.—*Object of Present Experiments.*

The present investigation arose from the necessity of a screened local oscillation generator for use in continuous-wave reception on a system of wireless direction-finder. The local oscillations are most conveniently generated by a simple form of triode oscillator operating entirely from a 6-volt battery, and the problem is to protect the receiving coil system from induced local oscillations, which were to be led to the required point in the circuit by a purely conductive connection. The final form of screened triode oscillator may, however, be applied to other purposes than the above. It is, for example, a very necessary appliance for certain methods of measuring the strength of received radio signals.

The transfer of electrical energy from one oscillating circuit to another may take place by either pure mutual induction or by radiation, or by both these factors. In the case under consideration, in which the closed oscillating circuit of the generator is placed comparatively near the receiving circuit, the transfer is almost entirely by mutual inductance, since the radiation will be negligible by comparison. The problem may, therefore, be considered correctly as the limiting of the high-frequency magnetic field around the oscillator by induced eddy currents in the screening body.

3.—*Preliminary Experiments.*

Some early miscellaneous experiments serve to indicate the difficulty of constructing a perfectly screened oscillation generator. These experiments were carried out with a compact form triode oscillator, using a single R-valve operating entirely from a 6-volt battery, and having a wave-length range of from 1,000 to 10,000 metres. With this instrument and a large-frame coil receiver and a seven-valve amplifier in use at Teddington, the more powerful and near C.W. transmitting stations, such as Horsea and Chelmsford, give a heterodyne note which can be heard several yards away from the telephones, even when the oscillator is placed at the far end of the wireless hut, about 100 ft. away. This indicates how very small is the oscillating current required to give a beat note with incoming signals.

When the oscillator and battery were placed inside a thin copper box, with a loose-fitting copper lid, the induced oscillations in the receiving coil were considerably reduced and were only effective in producing a beat note when the oscillator was within 20 ft. of the coil.

When the copper box was replaced by a galvanised-iron bin, a similar reduction in the induced oscillations was obtained of about the same order. In both these cases it was observed that the placing of the lid on the box had a considerable effect in the reduction ; but the position of the otherwise open end of the box relative to the receiving frame appeared to be immaterial to the strength of the induced signals. A piece of fine brass wire gauze was not so effective as a lid as a solid sheet of material, whether copper or iron.

To obtain much greater sensitiveness in the later experiments and also for more convenience when no suitable incoming signals were received, a second valve oscillator was used, placed inside or near the large receiving coil which was also tuned to the oscillator frequency, thus producing a constant beat note in the telephone receivers when any appreciable oscillation was induced by the generator under

experiment. The large receiving coil with tuning condenser and seven-valve amplifier thus acted as an extremely sensitive search-coil method of detecting the presence of any radiated or induced radio-frequency field.

The former oscillator and its battery was next placed inside the copper box, and this in turn was supported on paraffin blocks inside the iron box and with the respective lids in position. In this condition the beat note was only appreciable when these boxes were placed within about 12 ft. of the coil. By using a small coil in the amplifier circuit as a search coil, it was found that a considerable increase in the strength of the note was obtained with this coil placed near the aperture between box and lid, indicating that some, at least, of the energy escaped in this direction. Making metallic connection between the copper and iron box had no effect on the strength of the received signal.

Further experiments showed that the copper box provided with a well-fitting lid giving about 3 in. overlap was much more efficient in screening than with the lid formerly employed, only $\frac{1}{2}$ in. deep.

4.—*Experiments on Boxes with Multiple Copper Linings.*

As the combined copper and iron boxes seemed to give hopeful results, as above mentioned, further experiments were carried out with boxes with two and three linings.

One such box was made up as shown in Fig. 1, consisting of a wooden box *A*

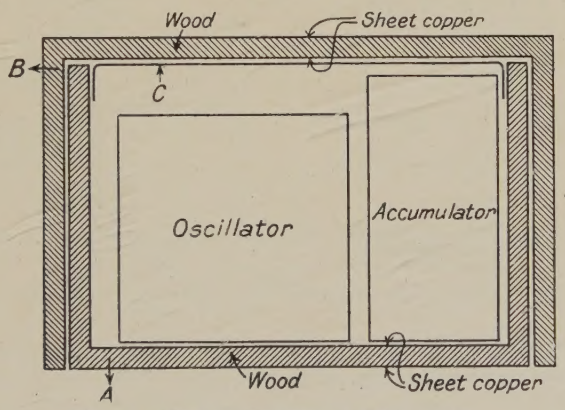

FIG. 1.—SIMPLE SCREENED OSCILLATOR.
A = Double-Copper-Lined Wooden Containing Box.
B = ,, ;, ,, ,, Lid.
C = Inner Copper Lid.

lined inside and outside with thin sheet copper, and supplied with a completely enveloping lid *B*, similarly double lined.

With this box it was found that putting on the lid made a considerable reduction in the escaping energy from the oscillator as indicated by the diminution in intensity of the note heard in the telephones. This box appeared, in fact, to be about as good as the combined copper and iron boxes previously tried, but was much more convenient from the point of view that the double-lined lid can be removed at one operation. The diminution effect was continuous as the lid was placed over the box, being a minimum when the box was completely enveloped.

It was subsequently found that the fitting of an auxiliary inner lid, as shown at *C*, effected a still further reduction in the emitted energy.

Another arrangement consisted of two copper boxes, complete with close-fitting lids, separated from each other by 1 in. wooden strips. This arrangement was found to screen most efficiently when the inner box was inverted, as shown in Fig. 2, when it was about the same as the last arrangement used. Again, no difference was experienced on making contact between the two boxes.

At this and subsequent stages of the experiments the need was felt for some simple and fairly accurate means of obtaining a measure of the emitted energy from such a type of partially-screened oscillator. What is essentially required is to measure the comparative strengths of the induced energy with the unscreened oscillator, and with the latter enclosed by the various screening arrangements which were used during the present investigation. The shunted telephone method might possibly be used, but the intensity of the note heard in the telephones does not necessarily indicate the amount of energy intercepted by the frame search coil, particularly when the oscillator is unscreened and the induced E.M.F. very large. It was often noticed, for instance, that even when the oscillator was partially screened, the intensity of the note in the telephones increased to a maximum

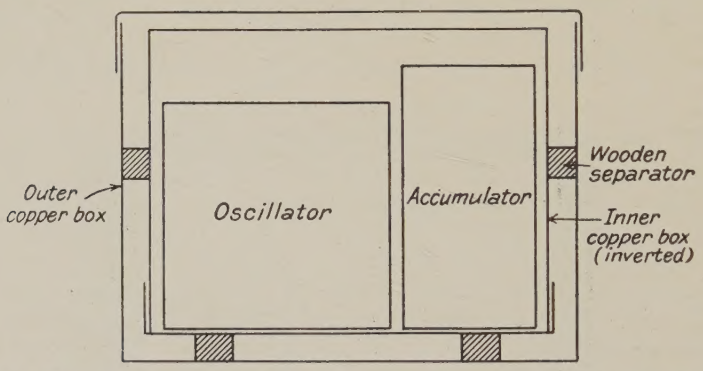

FIG. 2.—TRIODE OSCILLATOR, SCREENED WITH TWO COMPLETE COPPER BOXES.

before diminishing, when the oscillator was moved away from the coil. Also the interpretation of the results in terms of the distance of the oscillator from the frame is not simple.

It is possible that a somewhat elaborate arrangement would meet the requirements, involving a very constant amplifier and a means for calibrating it, but this was not considered to be within the scope of the present investigation.

5.—*Effect of Bringing Leads out from Screening Box.*

With a type of double-screened box as used above, the external field, therefore, could be rendered inappreciable when it was at a greater distance than 12 ft. from the search coil. In this condition a small coupling coil of a few turns was introduced into the inner copper box, and a flexible conductor brought outside both boxes. So long as this conductor remained entirely within the outer iron box, the beat note was undetectable, but immediately a few inches of it were exposed outside the iron box, even with a closely-fitting lid, good beat note signals were immediately obtained, giving evidence of energy thrown off from these leads. Replacing the

flexible wires by a lead-sheathed twin conductor overcame this trouble, which, however, recurred on connecting a small coil to the end of the screened leads, with a view to coupling on to the amplifier. By varying the coupling to the oscillator, this latter " radiation " could be reduced to an extent inappreciable compared with the oscillating energy which could be conveyed through the outer coupling to the amplifier.

6.—*First Experimental Screened Oscillator.*

Embodying the essential results of the experiments carried out as above, a first experimental oscillator was made up to the author's design by Messrs. H. W. Sullivan. This was in principle, although not in detail, similar to the arrangement shown in Fig. 3 below, consisting of a wooden box double lined with sheet copper 1/64 in. thick, fitted with an inner copper lid and with a second enveloping lid similarly

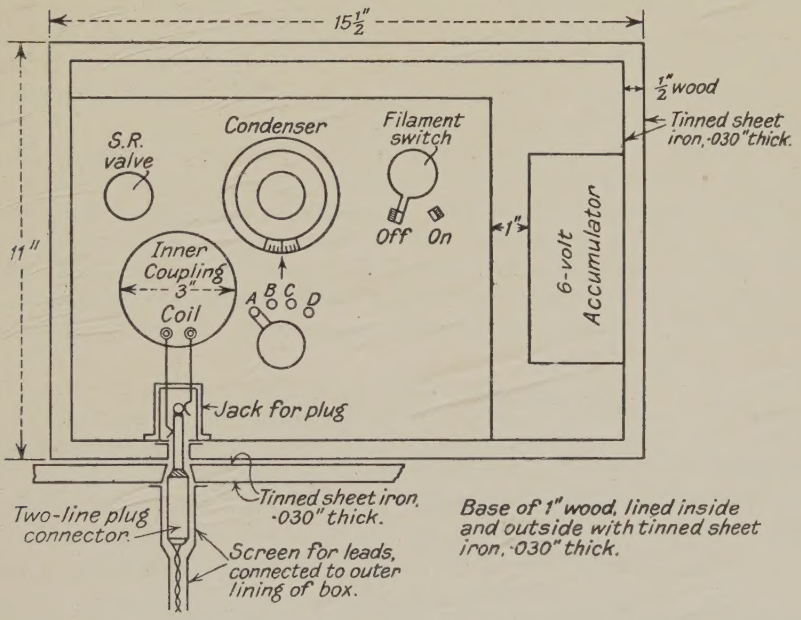

FIG. 3.—PLAN OF INTERIOR OF SCREENED OSCILLATOR.

constructed of wood and double lined with sheet copper. Ebonite connecting pieces were used between the control handles on the top of the lid and the variable condenser and switches for filament current and inductance tappings within the box. The metal parts of the control handles were arranged to make spring contact with the outer coating of copper.

A separate coil was used within the box to couple to the oscillator inductances, and leads from this coil were brought out through the side of the box by a simple telephone jack and plug connection. In the present arrangement, the jack is mounted inside the box and the plug is pushed through holes in lid and box which are only in alignment when the lid is in its final position over the box. The plug is screened by a metal coating in contact with the outer lining of the box and the connections are taken through a metal-sheathed twin flexible conductor to a double-D-shaped coil, forming half of the outer coupling. Leads from the other pair of D-shaped coils are taken to the output terminals mounted on ebonite at the side of the box.

The following points may be emphasised in regard to this screened oscillator arrangement :—

1. The entire control of the oscillator is obtained from outside, and it need never be opened except for inspection, replacement of valve, or recharging of accumulator. A valve (type SR) consuming only 0·1 ampere filament current is used, which with the accumulator fitted will give continuous operation for about 120 hours.

2. The opening up of the oscillator, when necessary, is a simple operation. It entails the removal of the connection plug, control handles and the two screening lids.

3. Fairly close metallic contact is obtained at all points in the paths by which the radio frequency energy may escape from the box, other than by direct penetration through the sides.

4. No part of the primary oscillator circuit is outside the inner metal lining. The secondary circuit is brought out by a screened conductor to an astatic coupling coil arrangement. The other half of this astatic coupling forms the tertiary circuit containing leads from the direction-finding coil to the amplifier. This coupling forms a convenient means of varying the strength of the local oscillation led to the receiver, varying from a maximum down to zero.

There is thus no part of any tuned circuit common to the inside and outside of the box, and all leads from the inside are carefully screened to reduce to a minimum any energy thrown off by these. With this oscillator in operation, but disconnected from the D.F. receiving circuit, beat-note action could just be detected on the latter, when the oscillator was about 5 ft. from the centre of the D.F. coil. In the case of the reception of a C.W. station, this heterodyne action was negligible, compared with the note intensity that could be obtained by connecting up the oscillator to the receiver in the proper manner.

As the pick-up effect from the oscillator by the large frame coil (5 ft. by 3 ft. 6 in.) is so small, it is reasonable to assume that that picked up by the amplifier will be negligible. The oscillator may, therefore, be placed close up to the amplifier to make the connections between the two as short as possible. As it is essential in the system to have the amplifier at least 5 ft. from the D.F. coil, the oscillator can conveniently be placed below or behind the amplifier and so be in a position in which its inductive or radiative action on the frame is quite negligible.

7.—*Use of the Experimental Screened Oscillator for Continuous Wave Direction-Finding Observations.*

With this first screened oscillator, used under the conditions outlined above, a number of observations were taken in duplicate on the few C.W. stations that can be received at Teddington with and without heterodyne. The results, all of which were made in the daytime, are summarised in the following Table :—

Station observed.	Wave-length km.	Without Heterodyne.		With Heterodyne.		Difference. A—B
		No. of observations.	Mean bearings. (A)	No. of observations.	Mean bearings. (B)	
Le Vallois ...	2·2	20	145°·0	18	145°·0	0°·0
Chelmsford	3·8	22	56°·3	24	56°·5	—0°·2
Horsea.....	2·5	22	220·5	22	220·8	—0°·3
,, ...	4·0	44	219·8	50	220·0	—0°·2
,, ...	8·0	Signals too weak		18	219·8	—

These observations show that on the stations observed the difference in the mean readings taken, with and without heterodyne, is usually negligible and never greater than the ordinary errors of D.F. observation. The normal position of the oscillator in the above results was 5 ft. west of the coil.

A number of observations were also carried out on Chelmsford, with the position of the oscillator varied, and these showed that no serious error is introduced until the oscillator is within 4 feet from the centre of the coil, in which position it is only about 1 foot from the side of the coil during part of its swing.

With the oscillator on the floor immediately underneath the D.F. coil, the error introduced was not more than 0·3°, while with the oscillator placed inside the frame, the error ranged up to 2° in either direction, according to the orientation of the oscillator relative to the frame.

No consistent difference in effect was obtained when the tertiary circuit of the oscillator was connected in either the grid or filament lead of the amplifier, but it may possibly help to balance up the " antenna effect " of the system as a whole if it is introduced into the grid lead.

As a typical example of the effect of " unscreening " the oscillator when connected up in its normal position the following result may be quoted: With the oscillator fully screened, the error introduced by it was about 0·1°; when the outer lid was removed, this error increased to 4·3°, and with the inner lid also removed, it became 8·3°.

Where it is possible to make the necessary connections, it would be preferable to connect the tertiary circuit of the oscillator in the grid circuit of the rectifying valve of the amplifier, to reduce the possibility of reaction from the high-frequency transformers.

8.—*Further Experiments on Screening.*

Using the above screened oscillator in conjunction with the coil receiving set as a very sensitive detector of a high-frequency magnetic field, a number of further experiments were carried out on the screening of the oscillator originally employed :—

(*a*) *Hermetically-sealed Copper Box.*

One of the original screening boxes was used, size 12 in. square by 8 in. high, made of sheet copper 25-30 mils thick. All joints at sides and bottom were sweated together with about $\frac{1}{4}$ in. overlap, the box being liquid-tight and probably gas-tight. A well-fitting lid was made for this of similar copper sheet, giving about $\frac{1}{2}$ in. overlap. This lid was actually used as the base with oscillator and battery placed on it, and the box over all. In such condition the external field was easily detectable by the receiving set when the box was within 20 ft. of it. A wave-length of 2,400 metres was used, as being fairly free from interference by external signals, and also giving greater possibilities of screening than a longer wave-length.

Without altering any of the adjustments the box and lid were solidly soldered up, using a good thickness of solder to cover up all holes and cracks. In this condition the effect of the syntoniser could still be detected when within 10 ft. of the frame. When the box was placed inside the frame the resultant beat note in the telephones could be heard 100 ft. away. Also by tilting the box inside the frame a position giving practically dead silence could be obtained in much the same way as is obtained with the oscillator without the screening box. No effect could be obtained by removing the box from the frame and placing it near the amplifier.

The effect of hermetically sealing up this copper box is, therefore, to reduce some of the emitted energy which presumably came from the cracks round the lid, while still leaving a very appreciable amount, apparently passing through the metal, the external field retaining the characteristics of that from the unscreened oscillator.

(*b*) *Hermetically-sealed Iron Box.*

A similar experiment to the above was carried out with a box of the same dimensions constructed of tinned iron sheet 28 mils in thickness.

With the oscillator placed inside the box, with a well-fitting lid giving $\frac{3}{8}$ in. overlap, the resulting effects of beat note were about the same as for the copper box. Placing another iron covering over the sides of the box made no appreciable difference, but when the lid as well was covered the emission was considerably reduced. This seemed to indicate that the major portion of the energy from the inner box was coming from under the lid.

Using the same wave-length of 2,400 metres, the box and lid were solidly soldered up as before, after which no beat note was audible in the telephones. The fact that the oscillator was still in operation undisturbed was proved by making a slit in the box, when a beat note was easily heard in the telephones. A subsequent repetition of this experiment showed that enough energy escaped through a slit $\frac{1}{2}$ in. long to be easily detected. Further experiments indicated the absolute necessity of stopping all holes and cracks and of getting good metallic contact across all the joints, as a "dry soldered" joint may easily be the cause of an external magnetic field.

These experiments indicate therefore that it is only possible to screen a valve oscillator completely, as far as the sensitivity of the above apparatus goes, by placing it inside an hermetically-sealed box of tinned sheet iron of sufficient thickness to prevent the direct penetration of the high-frequency magnetic field through it. In the latter respect iron would appear to be far superior to copper, which is a result in complete accordance with the theory of the penetration of alternating currents into conductors. (Skin effect.) This theory indicates that iron should be equivalent to about four to six times its thickness in copper. It is, therefore, probable that if experiment (*a*) above were repeated using a box of $\frac{1}{8}$ in. copper sheet, complete screening would also have been obtained.

(*c*) *Mercury-sealed Boxes.*

The process of soldering and reopening metal boxes containing a valve oscillator is very laborious and considerably limits experiments on these lines. It was, therefore, thought that a mercury seal, if of sufficient thickness, would be as efficacious as solder in stopping up holes and cracks.

A suitable tray $2\frac{1}{2}$ in. deep was therefore made on which either the copper or the iron box could be placed giving an all-round clearance of about $\frac{1}{2}$ in. Placing oscillator and battery in this tray with the iron box over it, the external field was easily detected when within 20 ft. of the frame. Filling up the tray with $\frac{1}{2}$ in. of mercury to close all cracks entirely stopped all detectable field, even when the box was placed inside the frame. On raising the box at one side the smallest crack was immediately detected by the note in the telephones.

Using the copper box in the above manner, however, a beat note was still obtained when the mouth of the box was entirely below nearly 1 in. of mercury. This comparison experiment is very striking. With the copper box placed over, the beat

note steadily diminishes, and a just detectable drop in intensity is obtained when contact is made with the mercury and the last crack closed up. With the iron box, however, the drop in note intensity to entire inaudibility when contact is made with the mercury is very sudden and a most marked effect.

These experiments were repeated for wave-lengths of 5,000 and 9,000 metres, and the residual effect with the copper box was slightly more noticeable on the latter wave-length, although the initial note-intensity was less due to the inefficiency of the amplifier at this wave-length. While these results indicate the conditions for effective screening over the range 2,000—10,000 metres, the problem is probably much simpler for shorter waves, but somewhat more difficult for longer waves. A slight increase in thickness of the box would, however, probably be effective up to the greatest wave-lengths at present in use. These results thus confirm fully those previously obtained and indicate that the most complete method of screening a valve oscillator is to enclose it in an iron box of sufficient thickness for the frequency used, and to seal hermetically all joints either with solder or mercury. Where very complete screening is required, therefore, this may be obtained on the above lines, using a plain iron box standing over an iron tray with welded joints, since mercury readily amalgamates with tin and solder. No external controls may be permitted, owing to the aperture necessary for these, but it would not be very difficult where an approach to ideal conditions is very essential, to remove the box when adjustments are required to be made. One serious disadvantage to this, however, is that the frequency generated by the valve set alters considerably when the box is placed in position over the oscillator. In all the above experiments, this fact always necessitated the continual readjustment of the auxiliary oscillator to retain the beat note within the audible limits.

9.—*Design of Screened Heterodyne for D.F. Sets.*

The general result of this investigation is that for applying auxiliary oscillations for C.W. reception on the D.F. directional wireless receiver in use, *an almost completely* screened oscillator, similar to the experimental sample used above, is suitable, with certain modifications. These consist in a reduction in the size and weight of the instrument, with the substitution of tinned-iron plate for the copper sheet previously used.

The details of the design of the instrument are shown in the accompanying Figs. 3, 4 and 5, which indicate the essential parts of the oscillator and screening box, to the scale of the modified design.

Fig. 3 is a plan view of the interior of the box, showing the positions of the oscillator and battery, with the various switches ; also the position of the inner coupling coil and the plug connection. Fig. 4 shows the positions of the screening lids with the method of obtaining external control. The exterior of the top of the box is illustrated in Fig. 5, showing the position of the D-shaped coupling coils.

10.—*Variation of the Screening Effect with Frequency.*

When a sheet of metal is placed in an alternating magnetic field, eddy currents are set up in the conductor, the field from which will tend to counteract the original magnetic field. The penetration of these eddy currents into the conductor will be governed by the ordinary well-known " skin-effect " formulæ, and hence the latter will determine the fraction of the original magnetic field which penetrates to any

depth in the conductor. The decay factor governing the amplitude of the high-frequency current at a depth "x" below the surface of the metal is $e^{-2\pi\sqrt{\mu f/\rho}\,.\,x}$,

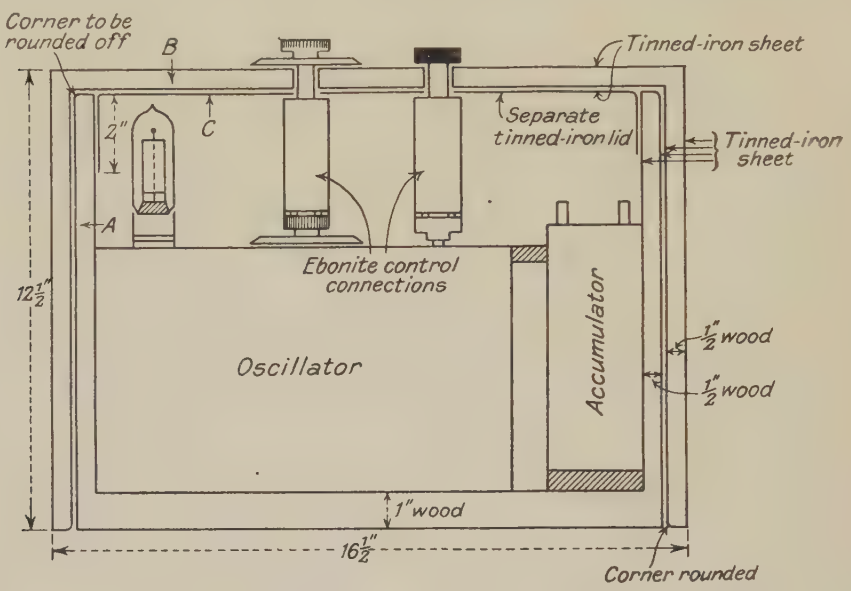

FIG. 4.—SECTIONAL ELEVATION, SHOWING TINNED-IRON SHEET LININGS, AND CONTROL HANDLES.

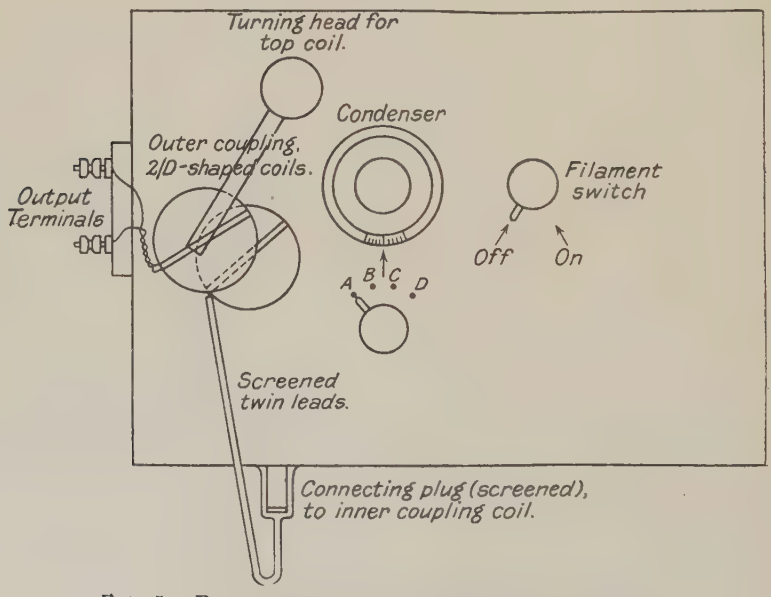

FIG. 5.—PLAN OF EXTERIOR OF SCREENED OSCILLATOR.

where "μ" and "ρ" are the permeability and resistivity respectively of the metal and "f" is the frequency of the alternations.

In the present case, this indicates that the field at a depth "x" is reduced to the fraction $a=e^{-2\pi\sqrt{\mu f/\rho}\,.\,x}$ of its original value at the surface of the metal. Hence, if the strength of the original magnetic field and the limiting value to which this must be reduced be known, the thickness of the metal required for the screen can be calculated.

In the absence of knowledge of the absolute magnitude of the "screened" field the comparative dimensions of the screens for different metals and frequencies may be obtained. For example, for a given frequency the thickness of iron and copper to produce the same values of "a" above may be obtained.

For copper $\mu=1$ $\rho=1,600$

,, iron $\mu=300$ $\rho=16,000$ approximately.

Hence

$$\frac{x_{\text{copper}}}{x_{\text{iron}}}=\sqrt{\frac{1,600}{1}\times\frac{300}{16,000}}$$

$$=\sqrt{30}$$

$$=\ 5{\cdot}5$$

i.e., it requires 5·5 times the thickness of copper to screen as effectively as a given thickness of iron.

The above expression also shows that the thickness of metal required to produce a given value of "a" varies as $\sqrt{\frac{1}{f}}$. In screening a circuit from a stray magnetic field, however, the usual condition is that the induced E.M.F. in the circuit shall be reduced below a certain limiting value. Now the E.M.F. induced in a coil by an alternating field of strength $H \propto f\times H$. If the circuit is now shielded by a metal sheet, the induced E.M.F. becomes

$$E=f\times e^{-2\pi\sqrt{\mu f/\rho}\,.\,x}\times H$$

which is the true equation giving the variation of the screening effect with frequency.

Thus, suppose it is required to determine the comparative thicknesses (x_1, x_2) of a metal necessary to give the same screening effect at frequencies of 100 and 1,000,000 cycles per second. In this case we have

$$10^2\times e^{-2\pi\sqrt{\mu 10^2/\rho}\,.\,x_1}=10^6\times e^{-2\pi\sqrt{\mu 10^6/\rho}\,.\,x_2}$$

$$e^{-2\pi\sqrt{\mu/\rho}\,.\,x_1}=10^4 e^{-2\pi\sqrt{\mu/\rho}\,.\,10^3 x_2}$$

$$-20\pi\sqrt{\frac{\mu}{\rho}}x_1=\log_e 10^4-2000\pi\sqrt{\frac{\mu}{\rho}}\,x_2$$

For copper, $\mu=1$ and $\rho=1,600$; hence

$$\frac{\pi}{2}x_1=50\pi x_2-2{\cdot}3\times 4$$

$$x_1=100x_2-\frac{18{\cdot}4}{\pi}$$

$$x_1=100x_2-5{\cdot}86.$$

Thus, if a thickness of copper $x_2=\frac{1}{8}''=0{\cdot}32$ cm. is required to give effective screening

at 10^6 frequency, then at a frequency=100 cycles per second the thickness required to give the same screening effect is

$$x_1 = 100 \times 0{\cdot}32 - 5{\cdot}86$$
$$= 32 - 5{\cdot}86$$
$$= 26{\cdot}14 \text{ cms.}$$

This example would appear to demonstrate that the difficulties of screening are greatly enhanced at frequencies well below the radio limit—*e.g.*, within the range of audible frequencies, as used in many forms of inductance and capacity bridge measurements.

Since this Paper was written I have learnt that some experimental work on screening has been carried out at the R.A.F. Instrument Design Establishment, Biggin Hill, and at the Army Signals Experimental Establishment, Woolwich; but no details of this have yet been published. The results of the work at Biggin Hill are already being used, however, in the design of wireless instruments.

DISCUSSION.

Mr. MORTON: On behalf of the staff at Biggin Hill, I should like to express our appreciation of the author's work, and of the reference to our own with which his Paper concludes. The errors in wireless direction finding which are produced by adventitious inductive coupling with the local oscillator were fully recognised during the war. They were counteracted to some extent by fixing the oscillator to the rotating coil, and adjusting its position in such a way as to minimise the evil. Experiments on screening were carried out in 1918, but this plan was not adopted in practice till February, 1921, when devices somewhat similar to those described by the author were brought into use.

Dr. J. H. VINCENT: The author has called our attention to the surprisingly large amount of energy that can escape through a crack in the screen surrounding an oscillator. A similar fact was proved in the case of sound by Lord Rayleigh; and in experiments on the screening of sound carried out some years ago, I found that at a pitch such as middle C a quite considerable amount of energy can get through a crack measuring, say, $\frac{1}{8}$ in. by 1 in. in a solid screen. It might be interesting to design an oscillator calculated to give out a minimum of energy by induction or radiation. By constructing the inductances and condensers in a suitable manner, it might be possible to confine the bulk of the emitted energy to the purely conductive path.

Mr. F. E. SMITH: I might remark that the author's research was undertaken at the instance of a Committee of the Radio Research Board, which is investigating the errors of wireless direction-finders. Observations made at some twelve stations throughout the country indicate that while considerable accuracy is attainable with damped waves, undamped waves give results which are vitiated by the radiation of energy from the oscillator. The problem of screening the latter has, I think, been satisfactorily solved by the author. As regards the calculation at the end of his Paper, the value of 300 assumed for the permeability of the iron seems rather optimistic, considering the low value of the magnetic fields involved.

Dr. A. RUSSELL: The author's formulæ for the skin effect apply to the case where the magnetic field is at right angles to the screen. Has he considered the case where it is parallel to the screen?

THE AUTHOR: I must thank Mr. Morton for his kind reference to the work described in the Paper. I am interested in Dr. Vincent's reference to the work of Lord Rayleigh on the escape of sound through cracks. One is reminded of the experience on closing a window; it is the last fraction of an inch that makes all the difference in excluding sound. The ordinary oscillator gives out about a million times as much energy as is required, and the trouble is that no substantial decrease can be obtained without stopping the oscillations altogether. Possibly the use of a toroidal inductance or something of that sort would reduce the external magnetic field to within reasonable limits. In reply to Mr. Smith—I had tried to discover an established value for μ at low values of H; but the results given by different experimenters differ widely, and the figure given in the Paper claims to be nothing more than a rough approximation to a mean value. In reply to Dr. Russell—I take it that the skin-effect formula applies to the magnetic component parallel to the screen, while the perpendicular component is mainly responsible for the large effect of a crack. Assuming that this latter component is normally screened by the circular eddy-currents which it sets up, we may suppose that a crack, by distorting the current flow, becomes subject to an oscillating potential difference between its ends, and so acts like a small Hertzian oscillator.

XX. *Viscosity Determination by Means of Orifices and Short Tubes.* *By* W. N. Bond, *M.Sc.(Lond.), A.R.C.S., A.Inst.P., Lecturer in Physics, University College, Reading.*

Received January 10, 1922.

ABSTRACT.

The Paper consists of an investigation of the corrections applicable to the determination of the viscosity of liquids by flow through tubes, due to the abnormal flow at the ends.

General *expressions for the end-corrections* are obtained by the method of dimensions. These are employed in plotting the results obtained from experiments on the flow of mixtures of glycerine and water through pairs of tubes of the same diameter, but of different lengths.

The conditions that the flow at the ends may be purely viscous, and the value of the end-correction in this case, are obtained from the present experiments, and also from experiments on an orifice described in a previous Paper.

Equations are given enabling viscosity to be measured by experiments on the flow through orifices or short tubes.

Introduction.

This Paper consists of a full investigation of the allowance that must be made for the end-effects when measuring viscosity of liquids by flow through a capillary tube. The results enable short tubes or even cylindrical holes in plates to be used for measurements of viscosity, without previous calibration by means of a liquid of known viscosity.

In order to make the end-effects definite and independent of the surface tension, the liquid was not allowed to emerge into the air on leaving the capillary tube, but passed into a second vessel of liquid. The tube used was thick-walled, and the two ends were ground off at right angles.

The abnormal conditions at the ends cause the pressure-difference between the liquid in the two vessels to be greater than the value deduced theoretically without allowing for these end effects. The end correction usually applied is proportional to the square of the rate of flow, and is due to the kinetic energy of the liquid. The value of this correction has been for a considerable period in doubt; and even recently complete unanimity of opinion did not exist.* Hence it was thought desirable to make determinations in the present experiments.

For any particular liquid and tube, for rates of flow less than a certain critical value, the flow at the ends will be purely viscous; and the end-correction under these conditions will be proportional to the rate of flow. It has been mentioned by various writers that such a viscous end-correction must exist, but no numerical data are given. This correction is investigated theoretically and experimentally in the present Paper, one estimate of its value being obtained from the author's experimental results on viscous flow through an orifice.†

* Edser, "General Physics for Students," 1913, p. 498.

† Proc. Phys. Soc., Vol. XXXIII., Part IV., pp. 225-230 (1921).

General Theory of the End-Corrections.

Consider two tubes of lengths l_1 and l_2, both having an internal radius r. When the rate of flow is in each case Q c.c. per second, let p_1 and p_2 be the total differences of pressure tending to cause flow in the two cases.

($p_1=h_1 g \rho$, where
h_1=difference in level between the exposed liquid surface in the two vessels
g=acceleration due to gravity
ρ=density of liquid.)

The ends may be supposed not to affect appreciably the flow at parts of the tube more than a certain distance from them. Then it follows that

$$\left.\begin{aligned} p_1 &= l_1 \cdot dp/dl + p_e \\ p_2 &= l_2 \cdot dp/dl + p_e \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (1)$$

where dp/dl denotes the pressure gradient causing flow at parts of the tube distant from either end; and p_e denotes the excess of pressure difference causing flow over that required to cause an equal flow if there were no end-effects. It will be noticed that p_e refers to the combined effects of the two ends.

From equation (1) it follows that

$$\left.\begin{aligned} dp/dl &= (p_1-p_2)/(l_1-l_2) \\ p_e &= (l_1 p_2 - l_2 p_1)/(l_1-l_2) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (2)$$

which enables dp/dl and p_e to be deduced from the pressure differences required to cause an equal flow through two tubes of the same internal diameter but different lengths.

General equations relating the end-correction p_e with the other variables concerned may be obtained by the method of dimensions. Let μ and ρ denote the viscosity and density of the liquid, and Q the volume passing per second. All the variables concerned may be related by the equation $\varphi_1(Q, r, p_e, \mu, \rho)=0$, where φ_1 is a non-dimensional function. Since there are only two independent non-dimensional products of the five variables, we may write without loss of generality

$$\varphi_2(Q\mu/p_e r^3 . \quad Q\rho/r\mu)=0,$$

and hence

$$p_e = Q\mu/r^3 . \varphi_3(Q\rho/r\mu) = Q^2\rho/r^4 . \varphi_4(Q\rho/r\mu) \quad \ldots\ldots\ldots \quad (3)$$

In a similar way, for parts of the tube distant from either end,

$$dp/dl = Q\mu/r^4 . f_3(Q\rho/r\mu) = Q^2\rho/r^5 . f_4(Q\rho/r\mu) \quad \ldots\ldots\ldots \quad (4)$$

When the pressure differences are independent of the viscosity, f_4 and φ_4 are constant. Under these conditions the flow is generally described as "turbulent." Similarly for purely viscous flow f_3 and φ_3 are constant, the pressure difference being independent of the density.

If $1/\sqrt{\varphi_4}=Q/r^2 . \sqrt{\rho/p_e}$ be plotted against $\sqrt{Q\rho/r\mu}$, it is easy to show that the curve will be linear both in the region where the flow is purely viscous, and where it is purely "turbulent." This method of plotting is employed in this Paper.

Description of Experiments.

To determine values of p_e, a pair of uniform thick-walled glass tubes of the same internal diameter, but of different lengths, was used. The difference in level between the surface of the liquid in the two vessels was determined for the same

rate of flow in each tube. The method of experiment was almost identical with that used previously in experiments on an orifice.* Experiments were carried out with different pairs of tubes, the details of which are given below. The different sets of experiments are indicated by letters, which are also inserted by the corresponding sets of determinations in Fig. 1.

Experiment.	r	l_1	l_2	ρ	μ
(a)	0·135	0·626	2·605	1·250	6·3
(b)	0·1001	0·853	2·685	1·249	4·8
(c)	0·1001	0·853	2·685	1·227	1·067
(d)	0·1001	0·853	2·685	1·183	0·211
(e)	0·1001	0·853	2·685	1·158	0·106
(f)	0·1001	2·685	13·66	1·11	0·049
(g)	0·1001	2·685	13·66	1·05	0·022
(h)	0·1001	2·685	13·66	1·00	0·0101
(i)	0·1001	0·853	2·685	1·00	0·0094

Mixtures of glycerine and water were used for all these experiments. As some experimenters seemed to doubt whether the viscosity of very viscous liquids is independent of the rate of shear,† preliminary experiments were carried out using glycerine. The flow took place through a capillary tube, and the result of the experiments was that no variation in μ could be detected at 0°C. between rates of shear at the walls of 4·1 and 0·6 radians per second ; nor at 14°C. between rates of shear of 15·7 and 0·6 radians per second.

The experimental determinations of p_e are exhibited in Fig. 1, where the non-dimensional functions derived in the section on general theory are plotted. This figure will be considered in detail in the following section of the Paper.

Values of the Viscous and Turbulent End-Corrections.

The portion (*a*), (*b*), (*c*) of the curve shown in Fig. 1 corresponds to the flow, being purely viscous not only in the tube but at each end. Hence from equation (3) we have $p_e=(Q\mu/r^3)\varphi_3$, where φ_3 is now a constant. The total end-correction may be considered as increasing the effective length of the tube by an amount equal to k times the internal diameter of the tube. Hence

$$p_e=(16k/\pi)(Q\mu/r^3) \quad \text{or} \quad k=\varphi_3 \,.\, \pi/16 \;.\;.\;.\;.\;.\;.\;.\;. \quad (5)$$

Thus, it will be seen that k is an absolute constant when there is purely viscous flow at the ends, provided the ends are made of definite form as described previously.

The value of k or φ_3 for these conditions should be capable of derivation from the general hydrodynamical equations when only terms involving viscosity are of importance.

A fairly accurate estimate of k is obtained from the slope of the portion (*a*), (*b*) (*c*) of the curve in Fig. 1. This gives a value of $k=0{\cdot}566\pm0{\cdot}020$. (The comparatively large probable error is chiefly due to the fact that the quantity measured is only a correction factor.)

A second estimate may be deduced from the results obtained by the author when experimenting on a small orifice.* The thickness of the orifice l was equal to 0·051 times its diameter $2r$; and it was found that the orifice could be replaced (for purely

* Loc. cit.

† For references *see* the Author's paper on orifice flow, previously mentioned.

viscous flow) by an equivalent length, $2k'r$, of tube of radius r, where $k'=0{\cdot}631$. Hence, allowing for the effect of the thickness of the orifice plate, we have

$$k=k'-0{\cdot}051=0{\cdot}580\pm0{\cdot}01.$$

This is in good agreement with the other determinations given above.

Combining the two experimental determinations and using equation (5), we have, for purely viscous flow,

$$k=0{\cdot}573\pm0{\cdot}009 \text{ and } \varphi_3=2{\cdot}92\pm0{\cdot}05 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Hence, by equation (3)

$$p_e=(\mu Q/r^3) \; . \; (2{\cdot}92\pm0{\cdot}05) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The portion (*d*), (*e*), (*f*) of Fig. 1 corresponds to the flow in the tube being viscous, but the end correction being due to kinetic energy. Thus the flow on emergence into the second reservoir is probably turbulent. The ordinate marked *A* corresponds

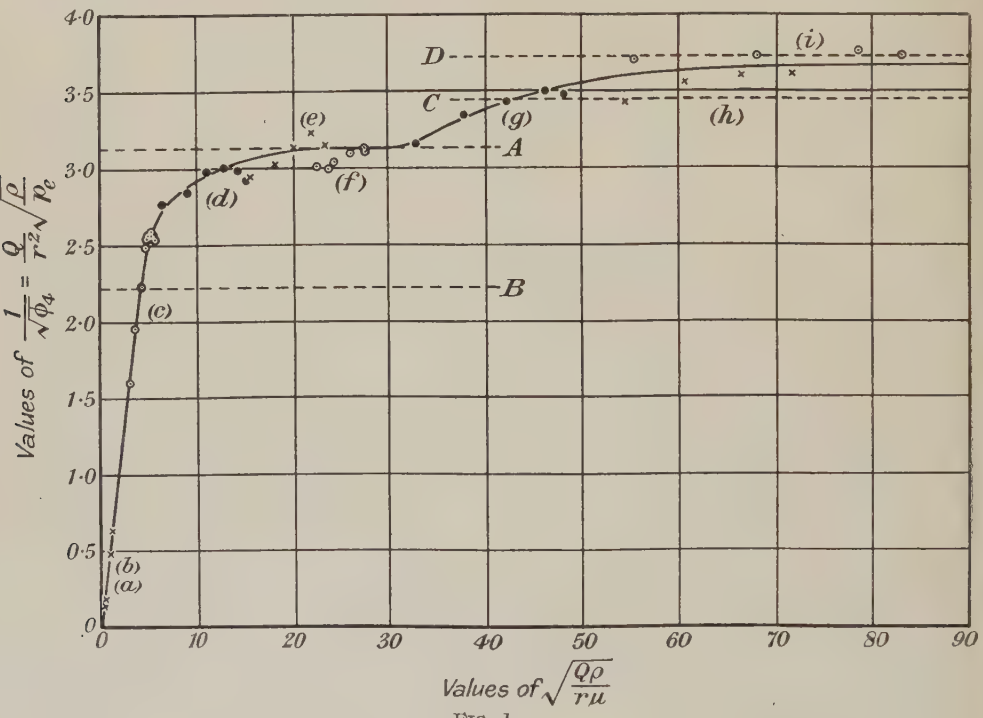

FIG. 1.

to the usually accepted value of the end correction under these conditions. This will be seen to be in close agreement with the present experiments. The ordinate *B* corresponds to the value given by Edser.*

The portion (*g*) corresponds to the stage when the flow in the tube is becoming turbulent, and the part (*h*) (*i*) to the flow being turbulent both in the tube and at emergence. The ordinates *C* and *D* give the range of the values usually quoted as applying to the end effect for large pipes. Since the flow is then turbulent, the agree-

* Loc. cit.

ment between these and the present experiments acts as a further confirmation of the present determinations.

The transition (*g*) may have occurred at lower rates of flow than usual in the present experiments, as all the tubes were short, and the edge at the inflow end was in no case rounded.

Application to Viscosity Determinations.

When a liquid is arranged to flow through a tube as described in the Introduction the value of the end correction p_3 can be obtained from Fig. 1 if the other variables be known. If μ be not known, an approximate value can be assumed and p_e deduced. This will then yield a closer approximation to μ, and so on.

It can be seen on inspecting Fig. 1 and the figures in the Paper on flow through orifices,* that the flow will be purely viscous at all points, provided

$$Q\rho/r\mu < 16 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Under these conditions the viscosity is given by

$$\mu = \pi r^4 p/8Q(l+2kr) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where $k = 0{\cdot}573 \pm 0{\cdot}009$.
l = length of tube
r = radius of tube.
p = total difference in pressure causing flow.
Q = volume of liquid passing per second.

This equation enables viscosity to be determined by purely viscous flow through quite short tubes or even orifices made by drilling a hole in a plate, l being in this case the thickness of the plate.

When $Q\rho/r\mu$ has values above 250, but the flow in the tube remains purely viscous, the end correction will be within $\pm$ 3 per cent. of the value of $p_e = \rho Q^2/\pi^2 r^4$ usually assumed.

For values of $Q\rho/r\mu$ intermediate between the two above cases, the value of p_e may be obtained from Fig. 1, as described above.

When the statement (8) is not true, it will in practice be necessary to use moderately long tubes in order to determine μ with any appreciable accuracy, for otherwise the kinetic end-effect may become larger than the main term being measured and, also, the flow in the tube may become turbulent.

In conclusion, the author would like to thank Prof. W. G. Duffield for the facilities that have enabled these experiments to be carried out; and Mr. J. S. Burgess, the laboratory steward, for his kind help in all experimental matters.

DISCUSSION.

Prof. A. O. Rankine: The author's investigation aims at showing that it is possible to measure viscosity by means of a short tube. This reverses the usual procedure of using a long tube in order to reduce error due to doubtful end-corrections. Is there any practical reason why a long tube should not be used?

Dr. J. S. G. Thomas inquired as to whether the magnitude of the end correction could even now be regarded as definitely settled. The matter had been discussed in a very recent Paper (Phil. Mag., Feb., 1922), in which it was also pointed out that when employing a variable head in

* Loc. cit.

viscosity determinations, it was not, in general, accurate to employ the mean value of the head for the purpose of calculating the absolute value of the viscosity. The author had employed a variable head in his experiments, and it would be interesting to know whether this possible source of error necessitated any correction. In the apparatus shown, the efflux of liquid took place into a body of the liquid under experiment, whereas in commercial types of viscometers, such as the Redwood, employing a short tube, the efflux occurred into air. A comparison of the results afforded by the Redwood and Engler types of instruments with those afforded by a method employing a long capillary has been made by Higgins (Collected Researches, National Physical Laboratory, Vol. XI., pp. 1-16), and it would be interesting to know whether the results of the author's experiments are in agreement with these results. The usual end-correction applied in the case of liquids was the entrance end-correction. In the case of the efflux of gases through orifices in plates, the phenomena at the exit end were of at least equal, if not more, importance. In the case of the discharge of air through an orifice into the surrounding atmosphere, the relation between the pressure of discharge and the volume of air induced or set in motion per unit volume of the discharge was quite different over a certain region of pressure according as the orifice was one in a thick or thin plate. He inquired as to how far similar considerations might hold in the case of the efflux of liquids from long and short tubes.

The AUTHOR (partly communicated) : In reply to Dr. Rankine : The use of a short tube has advantages in the case of a very viscous liquid, since the apparatus can be very much smaller than would be necessary if the end-corrections were to be made negligible. It is thus much easier to keep the apparatus at a uniform constant temperature. In reply to Dr. Thomas's remarks : The author considers that the magnitude of the end-correction, under definite conditions mentioned in the Paper, is known to a fair degree of accuracy. On the other hand, a complete *theoretical* treatment may still be lacking. The Paper by Lidstone (Phil. Mag., Feb. 1922), which the author has now examined, gives the usually assumed value of the end-correction when a constant head is used. The deviation of this value, however, assumes that the viscous loss of energy and the kinetic energy removed from the system at emergence act independently. This leads to the conclusion, pointed out by Edser, that the pressure difference between the upper reservoir and the emergent liquid varies with the distance of the stream line from the axis of the tube (which is almost certainly erroneous).

Errors due to the variable head do not affect the present experiments. The author was aware of this matter and used a logarithmic method of plotting in the experiments described above, undertaken to test whether μ depended on the rate of shear. The author has not compared his results with any in which the liquid flows through an orifice or short tube *into air*. The results of such experiments might depend on whether the liquid wet the outer surface or flowed in a compact cylindrical stream. For purely viscous flow the end-correction p_e is half caused at entrance, and half at emergence. When the end correction is due to kinetic energy, an orifice does not satisfy the conditon that there shall be no overlap of the two end-regions. However, the figures in the present Paper and that on orifice flows referred to, may be compared to find the effect of the length of the tube on the end effect. The present Paper gives no direct information as to the amount of liquid that is set in motion in the lower reservoir. Moreover, experiments in which gases are used cannot be compared with those on viscous liquids unless the variation in density in the case of the gas is negligible.

XXI. *The Position of Best Focus in the Presence of Spherical Aberration.* By T. Smith, *F.Inst.P.* (*Optics Department, National Physical Laboratory*).

Received December 9, 1921.

ABSTRACT.

Focal variations of phase in the presence of spherical aberration are calculated directly from the axial intersection points of rays of known inclination, and the graphical method employed shows that the focus for which phase variations have a minimum value may be found and suitably interpreted when the finiteness of the wave-length is disregarded. The diagrams here employed for the representation of spherical aberration have advantages over the more usual diagrams in several important particulars, and it is suggested that the older forms should be abandoned.

The increased interest which has been taken in recent years in the basic principles underlying technical optical work appears to have led to wider acceptance of the view that geometrical reasoning leads in a number of important practical problems to conclusions at variance with those which would be deduced directly from a wave theory of light propagation. That this view is erroneous will be at once evident when it is remembered how immediately the geometrical theory follows from the postulates of the wave theory, provided it is always remembered that diffraction effects exist and are modified in the presence of large amounts of aberration. In

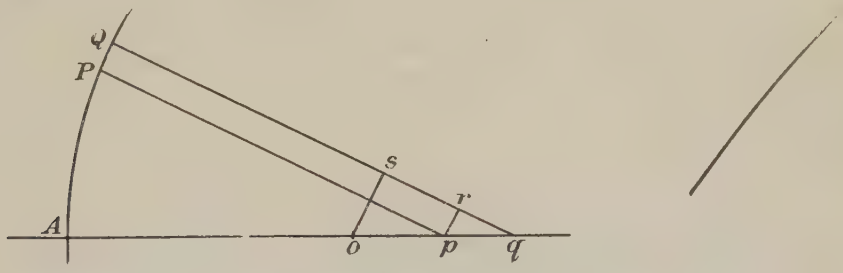

Fig. 1.—Determination of Phase Differences from Ray Positions.

cases where the defects are small, the diffraction system differs very little from the ideal pattern usually considered in mathematical discussions, and an optical instrument may attain its theoretical resolving power while very noticeable aberration is present. The spreading of the light patch due to diffraction and to aberration may therefore be regarded as independent effects, and when the latter is sufficiently large not to be obscured by the former the appearance of the image is in accordance with that predicted by geometrical reasoning.

A simple particular case of the supposed inconsistency is afforded by the best position of focus for the image of a point source formed by a symmetrical beam of monochromatic light when the wave front is not strictly spherical. The detailed discussion of this particular problem may, therefore, be of interest.

Suppose that APQ represents a section of the emergent wave front in a plane of symmetry, and that the normals to the surface at P and Q, two near points in this plane, meet the axis at p and q. If 0 is a fixed point on the axis through which the

paraxial portion of the wave front passes at time $t=0$, the disturbance from the paraxial part of the wave front will reach p and q at times $\frac{0p}{v}$ and $\frac{0q}{v}$, where v is the velocity of light. The rays from P and Q will, in general, pass through p and q respectively sooner or later than the paraxial disturbance. Let the times be $\frac{0p}{v}+\tau$, $\frac{0q}{v}+\tau'$. Since P and Q are near points on the wave front, the rays Pp and Qq make only a slight angle with one another, and the disturbance from PQ, therefore, reaches p and r simultaneously, where r is the foot of the perpendicular from p to Qq. The lag in time at q compared with p thus depends only on the time taken by the disturbance to travel from r to q, or

$$\frac{0q}{v}+\tau'=\frac{0p}{v}+\tau+\frac{pq\cos\psi}{v}$$

where ψ is the inclination of Qq to the axis. Putting $\tau'-\tau=d\tau$, and denoting the longitudinal spherical aberration by ξ, so that $pq=-d\xi$, this equation may be integrated in the form

$$v\tau=\int_0^{\xi}(1-\cos\psi)d\xi$$

The integral is to be evaluated from the known variation of the spherical aberration with the inclination of the emergent ray. It is to be noted that τ is simply the difference in the times of arrival at the common point of the corresponding rays of disturbances arising from an outer zone and from the paraxial portion of the wave front, and necessarily vanishes at the lower limit of integration. Assuming simple harmonic trains of waves of wave-length λ, the disturbances which arrive simultaneously will differ in phase by

$$\frac{2\pi}{\lambda}\int_0^{\xi}(1-\cos\psi)d\xi$$

It is not sufficient to determine phase differences only for the geometrical intersection of ray paths, but it must be possible to determine them for an arbitrary axial point in the neighbourhood of the intersections. Let 0 be such a point. The disturbance from Q will reach 0 and s simultaneously, where s is the foot of the perpendicular from 0 to Qq. The paraxial disturbance reaches 0 at time $t=0$, and the oblique disturbance at time $t=T$. The time of arrival of the oblique disturbance at q is therefore

$$T+\frac{0q}{v}\cos\psi$$

and comparing this with the expression found before

$$vT+0q\cos\psi=0q+\int_0^{\xi}(1-\cos\psi)\mathrm{d}\xi$$

or

$$vT=0q(1-\cos\psi)+\int_0^{\xi}(1-\cos\psi)d\xi$$

and from this the corresponding expression for differences of phase relative to any axial point at once follows.

The determination of phase differences from the calculated aberration is most simply effected by plotting the aberration against $1-\cos\psi$. The resulting curve is normally very smooth with slight curvature, and in a large number of cases differs imperceptibly from a circular arc. If AB represents such a curve where A is the paraxial focus and b the marginal focus, the difference of path at the paraxial focus is given in the same units as those used for the measurement of the aberration by the area ABb. At the marginal focus the difference will be represented by the area ABC, and the sign of the phase difference differs in the two cases since the areas lie on opposite sides of AB. At an intermediate axial point such as d where light comes to a focus from the zone ψ_0 of the wavefront where $1-\cos\psi_0=dD$, the difference of phase between this zone and the paraxial rays is given by the area AdD, and that between this zone and the marginal zone by the area BDe. There is evidently a point d between A and b which makes the two areas AdD and BDe equal in magnitude and opposite in sign. For this focussing position the paraxial and marginal zones give disturbances which are in the same phase, and when the total phase differences are small this is, with a smooth aberration curve, almost indistinguishable from the

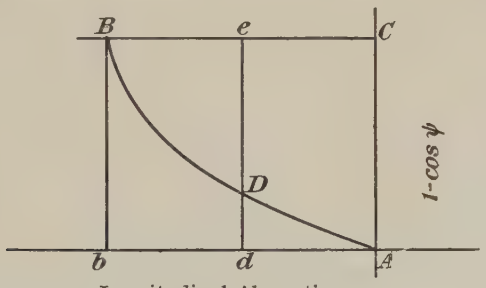

FIG. 2.—ABERRATION DIAGRAM FOR AN UNDERCORRECTED LENS.

position which results in the greatest axial amplitude of vibration, and in such cases is accepted as the position where the best images are formed.

When the aberration is not continuously increasing over the whole aperture the best focus will on this theory be found by an obvious modification, the total area enclosed between the curve and the ordinate on the one side being equal to the corresponding area on the other, as illustrated in Figs. 3 and 4.

Disregarding now differences of phase, the best mean focus which would be chosen in the presence of aberration on, say, a corpuscular theory would be the axial point at the centre of gravity of the ray-axis intersections. The proper weight is given to the zones for the determination of this point by plotting the longitudinal aberration against the cosine of the inclination of the ray to the axis. The curves obtained correspond exactly to those of figures 2 to 4, and the corresponding construction gives the same focussing point. The conclusion to be drawn is, that exactly the same position of focus follows from the geometrical considerations by which designers have been guided in the past as would be selected by taking into account the influence of phase discrepancies on the variation in light intensity as the tentative focus is moved along the axis.

A modified theory leads to curves in which the aberration is plotted against the square of the sine of the inclination. The curves found are similar to those described above with the single exception that they are slightly distorted in the direction

of the ordinates. If the same scale is used in the two cases for the paraxial region, so that the ordinates for the new construction are $\frac{1}{2}\sin^2\psi$, the ratio of the ordinates for the angle ψ in the two diagrams is $2 : 1+\cos\psi$. The effect of this distortion on the position of the ordinate which secures equal areas on the two sides is exceedingly small, and in typical cases that have been considered the difference in the best focus position by the two methods is at least as small as one thousandth of the longitudinal aberration present. Measurements of aberrations to such an accuracy or such precise

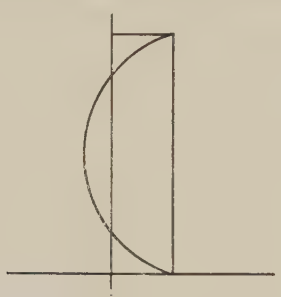

FIG. 3.—ABERRATION DIAGRAM FOR A CORRECTED LENS.

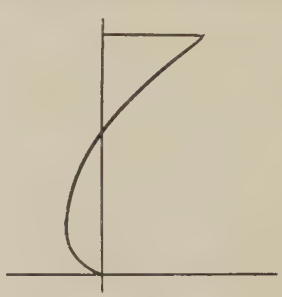

FIG. 4.—ABERRATION DIAGRAM FOR AN OVERCORRECTED LENS.

experimental settings as this difference implies are out of the question and indeed are physically without meaning. This close correspondence justifies the conclusion that any expressions for the aberrations which give reasonably accurate positions for marginal rays will be adequate for every other purpose.

The construction of diagrams in which the longitudinal aberration is plotted against $1-\cos\psi$ or $\sin^2\psi$ have many advantages over the customary diagram in which the ordinate is proportional to $\sin\psi$ or to $\tan\psi$ in addition to the ease with

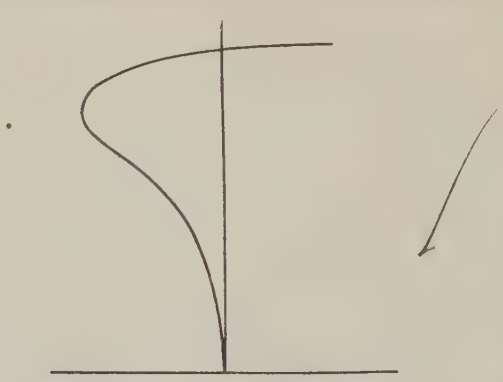

FIG. 5.—USUAL ABERRATION DIAGRAM SHOWING SLIGHT OVERCORRECTION.

which the position of best focus may be found. In the usual diagram aberrations of higher order tend to produce a curve in which changes of direction take place very suddenly, and it is difficult to draw a curve with accuracy connecting the calculated or observed points. A typical example is given in Fig. 5. It is difficult to find from such a curve with reasonable accuracy either the zone which sends a ray through the paraxial focus or the maximum amount of aberration present in intervening zones. The portion of the curve which occupies the greater part of the diagram

corresponding to rays near the axis is not in a form which affords any useful information. With the diagrams here advocated these objections do not hold. The curve is easily drawn with accuracy, the tangent to the curve at the paraxial point A gives the first order aberration coefficient, and the law of decrement of the aberrations of different orders is easily determined. In addition changes of shape of the lenses produce effects on the curve which may be utilised effectively in shortening calculations. Alterations of a very simple kind, for example, may be represented by rotating the curve without appreciable change of shape round the point A. These and other advantages are of sufficient importance to justify the complete abandonment of the older graphical methods of exhibiting aberrations.

The aberration known as coma may be simply discussed on similar lines and with similar diagrams to those here described for spherical aberration.

DISCUSSION.

Mr. J. GUILD: The position of best focus is usually defined in geometrical optics as that of the circle of least confusion, but the geometrical definition adopted by the author makes it coincide with the region of maximum density of the rays. This view is interesting in connection with modern theories of vision, such as that of Prof. Joly, of Dublin, according to which vision is due, not to a two-dimensional stimulation of the surface of the retina, but to a photo-electric effect which operates in depth throughout the volume of the visual purple. On this view the Author would be right in identifying the focus with the region of maximum concentration of energy. It is of interest to know what would be the difference in practical effect between the two definitions; it seems as though it might be appreciable in some cases.

Mr. F. TWYMAN: The search for a definition of the position of best focus arises from the desire to reduce to rigorous quantitative terms one's empirical estimate of the effectiveness of a lens. Unfortunately, those who have to use lenses will not be satisfied with a theoretically optimum design, but prefer that one of a number of lenses which seems to give the best results on trial. I am much struck by the last speaker's ingenious suggestion as to the volume-stimulation of the retina, but I doubt if there would be any appreciable difference in practical effect between the various definitions of focal length.

Lt.-Col. J. W. GIFFORD (communicated remarks): From the point of view of trigonometrical trace, I have long felt that the question of the best height from the vertex for the incident ray at the first surface on which successful calculations can be based much needs clearing up. Thanks are due to Mr. T. Smith for his able treatment of the subject from quite a new point of view. In reference to Mr. Smith's Figs. 3 and 4, and diagrams from the geometrical aspect of the same kind in some of the text books, I would like to ask him whether the patterns there shown apply when more than two optical media are used in the construction of the lens combination. That they represent the case for the first surface only, and in a modified form when more than one curved surface and more than one refracting medium exist in the combination, is doubtless the case. But my own experience with apochromatic triples and their trace has led me to think that the curves Mr. Smith has drawn would cross and re-cross the ordinate more than twice—indeed, in that case several times—so that there the problem becomes decidedly more complex.

A simple practical formula for approximately equal areas which I have found useful is

$$H=\sqrt{\frac{(R+t/4)^2}{2}}\Big/\frac{R}{h}$$

where h is the total height to the edge, t the thickness at the axis for that height, and H the height required. R is the radius of the first surface.

This formula gives a somewhat smaller value than Kerber's $H=\frac{h}{2}\sqrt{3}$, which is consonant with the smaller amount of light per unit area transmitted as we approach the edge.

From the AUTHOR (communicated): The remarks of all the contributors illustrate the absence of precise knowledge about the best focus from which we at present suffer. Theories may be constructed in large numbers which give answers for which much can be said, but every

one involves assumptions for which the experimental justification is yet lacking. When the necessary precise experimental observations are available it should be possible to say to what extent symmetrical optical instruments with surfaces of specified types can have their apertures and fields of view enlarged.

In reply to Col. Gifford, the author would generally prefer $H=\frac{1}{2}h\sqrt{2}$ to $H=\frac{1}{2}h\sqrt{3}$ to find the maximum spherical aberration when the full aperture is corrected, but no precision in the position is necessary. The methods described in the paper are applicable however many times the aberration curve crosses a given ordinate. In triple apochromatic objectives the author's experience is very limited, and those he has investigated in detail suffered from no aberration which would be likely to be measurable on physical grounds. Col. Gifford's experience with oscillating spherical aberration appears to accord with the author's.

XXII. *The Determination of Poisson's Ratio and of the Absolute Stress-variation of Refractive Index.* *By* F. TWYMAN, *F.Inst.P., and* J. W. PERRY.

RECEIVED FEBRUARY 18, 1922.

ABSTRACT.

The Paper describes a ready method of determining the stress-optical coefficients, the Hilger interferometer being employed. Young's modulus of elasticity and Poisson's ratio are determinable simultaneously.

AFTER the discovery by Brewster * of the phenomenon of double refraction in strained media, the first to make quantitative determinations of the effect was Neumann,† who also developed a theory of the phenomenon. Further experiments were carried out by Kerr, ‡ establishing the laws of the propagation of light in strained media. Absolute measurements were made later by Pockels,§ whose investigations resulted in the production of a glass which, when subjected to any strain within the elastic limits, remained optically almost isotropic. Filon‖ later investigated by various methods the birefringence and its dispersion in a bent beam, and advantageously proposed the use of two constants in specifying the stress-optical effect in such a beam when bent, called by him the stress optical coefficients, and defined as

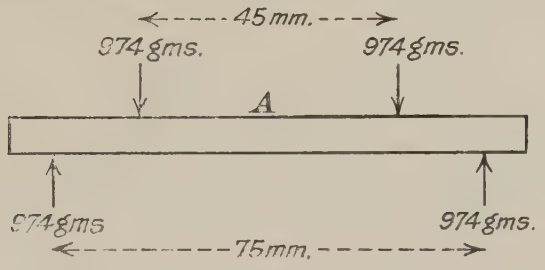

FIG. 1.

the " additional retardation per unit thickness per unit tension " for ordinary and extraordinary rays respectively. Filon specifies these constants in terms of " brewsters," a unit proposed ¶ also by him (1 brewster $=1\cdot10^{-10}$ cm.^{2}/gm.).

In the course of an investigation concerning the optical properties of a certain melting of glass, it became necessary to determine approximately the stress-optical coefficients. It is the purpose of the following to indicate the method employed, which differed from any known to have been published, and is applicable in connection with determinations of other small refractive index variations. ** No great accuracy is claimed for the measurements here recorded, but the method is susceptible of refinement.

A strip of the glass was prepared, of dimensions $80\times15\cdot1\times2\cdot48$ mm., of which the two $80\times2\cdot48$ mm. faces were accurately plane parallel, and one $80\times15\cdot1$ mm.

* Phil. Trans. (1816).
† Abh. d. Berliner Akad. (1841). II.
‡ Phil. Mag., 5, XXVI. (1888).
§ Ann. d. Phys., 4, 7, 745.
‖ Phil. Trans., A, CCVII. (1907) ; *see also* Proc. Camb. Phil. Soc., XI., 478, XII., 55, 314.
¶ Proc. Roy. Soc., A., LXXXIII., 572.
** As, for instance, the variations of thickness and of mean refractive index from point to point of nearly plane parallel glass plates.

face polished accurately plane. It was supported on two pieces of soft wire (electric fuse wire) 75 mm. apart, so that a load (of 1948 gms.) could be applied by resting a weight symmetrically on two similar pieces of wire 45 mm. apart, as is shown in Fig. 1.

Compressive stress at A : 94·66 kg./cm.^{2}.

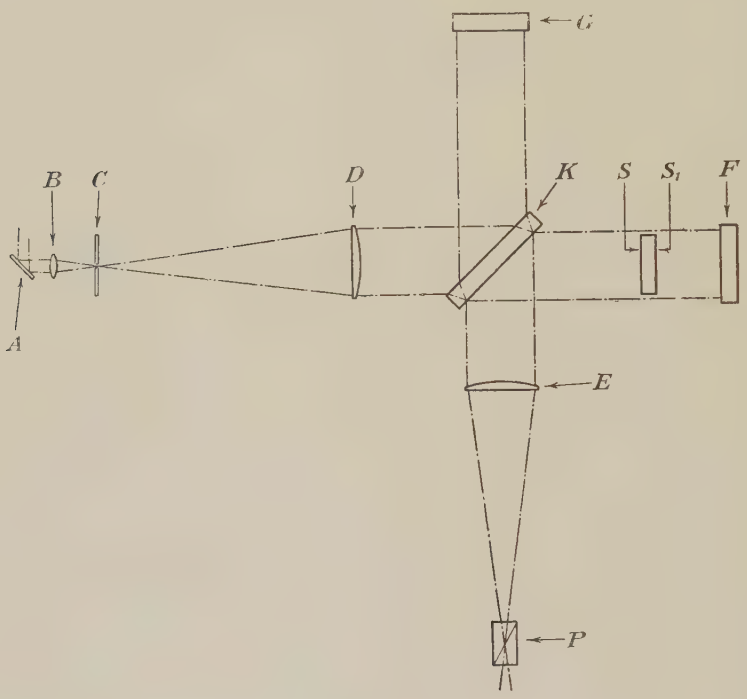

FIG. 2.

ARRANGEMENT (*a*).

The whole was placed in a Hilger prism interferometer* (Fig. 2), in the beam usually occupied by the prism, and the interferometer illuminated by the green Hg radiation $\lambda 5461 \mathring{A}$. U. The interferometer was so adjusted that uniform coloration was produced, with the strip unloaded, over the whole area presented by the strip.

On loading the strip interference bands as shown in Fig. 3 were produced. These are due to the composite effect of alteration in the length of glass path occasioned by the stress and variation of refractive index also occasioned by the stress. Since, moreover, the stress in general occasions double refraction, the appearance is a complex one, but it is readily simplified by admitting to the observer's eye light polarised either in a horizontal or in a vertical plane.

ARRANGEMENT (*b*).

Interference bands can also be obtained by covering the interferometer mirrors F and G and observing by means of the light reflected from the surfaces s and s' of the strip.

* Phil. Mag., VI., 35, 1.

If now n be the normal refractive index of the glass, $2t$ the length of glass path in the direction traversed by the light, Δn and Δt the variations in n and t due to loading the strip in the manner indicated, and m the number of complete bands observed from A to B (Fig. 3), to the approximation aimed at in the present determination we can write for the case of arrangement (*a*) :

$$\tfrac{1}{2}m_a\lambda=\Delta(n-1)t=(n-1)\Delta t+t\Delta n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

while for the case of arrangement (*b*) :

$$\tfrac{1}{2}m_b\lambda=\Delta nt=n\Delta t+t\Delta n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

from which two equations Δn and Δt are easily determinable.

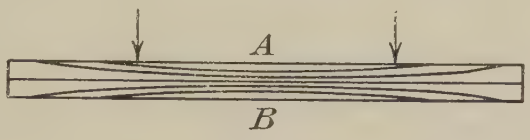

FIG. 3.

Particulars of the glass : Melt 683.* Derby Crown Glass Co.

$$n_D=1{\cdot}5759 \qquad \nu=41{\cdot}3$$

The following results were obtained :—

Arrangement.	Ordinary ray.			Extraordinary ray.		
	m_0	Δn_0	Δt	m	Δn_e	Δt
(*a*)	8	0·000110	1·65λ	4·3	0·000044	1·60λ
(*b*)	11·3			7·5		
Compressive stress at A ... ... 94·66 kg./cm.².						

Thus, for any known load, Δn for both ordinary and extraordinary rays is determinable immediately from the observations. It will also be noticed that Δt is twice determined. The two values found should, of course, be identical ; that they are not so is due to the fact that m was measured by counting the bands simply as seen by the unaided eye at P, the accuracy so obtained sufficing for the immediate purpose of the experiment. More accurate means of counting the bands will readily suggest themselves to one desiring to adapt the method to more precise determinations.

It would, of course, be necessary then to correct for several sources of error, which in the more refined application of the method would no longer be negligible, arising from the prismatic form of the strip when bent. It may be shown that to

* We have to thank the courtesy of the Derby Crown Glass Co., Ltd., for letting us have, and permitting us to publish, the theoretical composition of their melt 683 as determined from the batch used. No analysis of the actual glass was made.

Theoretical Formula :

100 SiO_2. 17 PbO. 11 Na_2O. 5 K_2O.

Theoretical Percentage Composition :

SiO_2	54·83
PbO	34·64
Na_2O	6·23
K_2O	4·30

the order of approximation aimed at in the present measurement such sources of error may be ignored (the greatest correction—*i.e.*, that to the value of $m_{ob}\lambda$, necessitated is approximately $0{\cdot}08\lambda$, and is thus smaller than the probable observational error).

The stress-optical coefficients as thus determined for the glass as above specified are as follows :—

$$C_0 = -5{\cdot}81 \qquad C_e = -2{\cdot}32$$

The above measurements enable us, since Δt is determined, to calculate Poisson's ratio if Young's modulus E is known for the glass. Failing a more precise knowledge of E it is possible to determine it on the same apparatus by applying the method originated by Cornu,* elaborated by Straubel,† and quite recently employed by Jessop ‡ in the determination of Poisson's ratio for specimens of plate glass. The procedure is as follows :—

A right-angled prism is so mounted under the loaded strip that light proceeding from the half-silvered mirror K (Fig. 2) is reflected normally from the under-surface of the strip, and passes back to the mirror K to be reflected to the observer's eye at P. The beam which is reflected from the curved under-surface of the strip interferes with the beam reflected from the upper surface of the prism ; the curvature of the strip can, therefore, be determined by counting the interference fringes so formed and observed by the eye at P. Because of the large number of fringes with the original load, the total load was reduced to 1·006 kg., thereby rendering possible a more accurate counting. The accuracy in E thus secured is estimated at about ± 3 per cent.

The following observations were made, y being measured from the centre of the strip along its major axis :—

y	7·5	10·0 mm.
$m\lambda$	6·75	11·75 λ

The radius of elastic curvature R is then given by

$$R = \frac{y_2^2 - y_1^2}{(m_2 - m_1)\lambda}$$

and Young's modulus E is calculated from

$$E = \frac{12\, RM_2}{td^3}$$

where M is the bending moment, the suffix being to distinguish this from the earlier value. E was thus found to be 6,320 kg./mm.^{2}.

Returning now to the observations firstly made, Poisson's ratio σ is to be derived from

$$\sigma = \frac{E}{12} \cdot \frac{d^2}{M_1} \Delta t \,;$$

thus is obtained the value $\sigma = 0{\cdot}196$.

DISCUSSION.

Mr. J. Guild inquired what fraction of a band could be counted in the determination of E.

The Author replied that the bands were counted to the nearest one-tenth.

* C.R., LXIX., 333 (1869).
† Wied. Ann., LXVIII., 369 (1899).
‡ Phil. Mag., VI., 42 (1921).

XXIII. *An Experimental Comparison of the Viscous Properties of (a) Carbon Dioxide and Nitrous Oxide; (b) Nitrogen and Carbon Monoxide.* *By* C. J. Smith, *B.Sc., A.R.C.S., D.I.C., Research Student, Imperial College of Science and Technology.*

Received February 24, 1922.

ABSTRACT.

The present Paper is the result of an experimental investigation to determine the viscous properties of (*a*) carbon dioxide and nitrous oxide, and (*b*) nitrogen and carbon monoxide. Direct comparisons have been made in each case by observing the time required by a mercury pellet to force an equal volume of gas through a capillary tube, at atmospheric and steam temperatures. For the above groups of gases the times of fall for each gas are equal at 15·0°C. and 100·0°C., and hence, independently of any knowledge of the absolute viscosity, it has been proved that the viscous properties are identical over the above temperature range. The absolute viscosity has been obtained by comparison with air, and the mean area of collision deduced by using Chapman's formula.

The Lewis-Langmuir theory of molecular structure has received much confirmation from the close degree of equality which exists between the physical properties of (*a*) carbon dioxide and nitrous oxide, and (*b*) nitrogen and carbon monoxide. In his Paper,* among other things, Langmuir showed, with the data then available, that the viscosities of nitrogen and carbon monoxide were approximately equal at 0°C., and proceeds to say: "This evidence alone should be sufficient to prove that the structure of the shell of the carbon monoxide and nitrogen molecules must be nearly the same." Similar considerations apply to the other pair carbon dioxide and nitrous oxide.

The mere fact that the viscosities of any pair of gases are equal at a particular temperature does not of itself support the view that the molecules in question have the same arrangement of electrons in the outer shell; for there are many cases of such equality in which there is no suggestion of molecular similarity: for example, the gases ethylene and cyanogen at 100°C., and ethane and hydrogen below 0·0°C. In order to infer from viscosity data identity of external molecular constitution, it is necessary to be able to show that the viscosities are equal at *all* temperatures, or, in other words, that the viscosity at any particular temperature and the rate of change of viscosity with temperature should be the same for each gas.

The object of the present research was to determine how nearly these conditions, are fulfilled by the two pairs of gases under consideration, *i.e.*, (*a*) carbon dioxide and nitrous oxide, and (*b*) nitrogen and carbon monoxide. Direct comparisons of the viscosities at both atmospheric and steam temperatures have been made in each case. The results support Langmuir's view to a much closer degree of approximation than can be derived from the figures which he himself quotes. It is found that nitrogen and carbon monoxide have the same viscosities both at 15·0°C. and at 100·0°C., to within two or three parts in a thousand.

Apparatus and Method of Observation.

The apparatus used has already been described elsewhere.† Only a brief description is necessary here. The viscometer is shown in Fig. 1, and it consists of a capillary tube and a piece of ordinary quill tubing which form a complete circuit.

* I. Langmuir, Journ. Amer. Chem. Soc., p. 904 (1919).

† A. O. Rankine, Proc. Roy. Soc., Vol. LXXXIV. (1910), and A. O. Rankine and C. J. Smith, Phil. Mag., Vol. XLII., p. 601 (1921).

This quill tube contains a pellet of mercury which forces the gas in the apparatus through the capillary tube. The apparatus was specially made for the purpose from tubes carefully chosen for uniformity and circularity of cross-section. The capillary tube had a mean diameter 0·19 mm., which did not vary by more than 1 per cent. in a length of 80 cm. The fall tube was about 0·3 cm. in diameter. Atmospheric temperatures were observed by means of a calibrated mercury thermometer

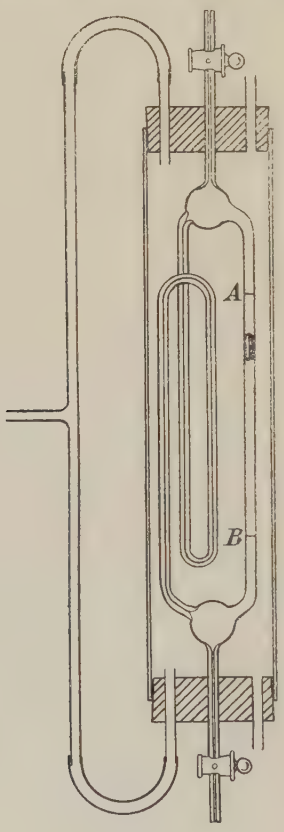

FIG. 1.

inserted in the jacket, and steam temperatures were obtained from observations of the barometer. The steam was admitted at both ends of the jacket and escaped freely.

In some recent experiments Professor Rankine and the author* found it necessary to modify the original method of observation in one important particular. It had always been fully recognised that the full hydrostatic pressure, due to the weight of the mercury pellet, was never operative on account of the capillary effects due to the difference in curvature of the ends of the mercury pellet. Evidence was then obtained that the capillary effect changed with the gas in contact with the mercury pellet, and with the temperature. It also changed slowly with time. These effects have been confirmed in the present experiments.

* A. O. Rankine, Proc. Roy. Soc., Vol. LXXXIV. (1910), and A. O. Rankine and C. J. Smith, Phil. Mag., Vol. XLII., p. 601 (1921).

The capillary effect can be eliminated almost, if not entirely, by making double sets of time observations; first with the mercury pellet intact, and then with it broken into two segments, each being approximately half the whole pellet. Experiment has shown that the capillary effect is doubled when the pellet is broken into two segments, and trebled when the pellet is divided into three parts. Let the capillary effect be denoted by x, *i.e.*, a certain fraction of the full hydrostatic pressure which would be operative if the capillary effect were absent; and let t_1 and t_2 be the observed times of fall between specified marks on the viscometer. Then an examination of the formula connecting the viscosity with the pressure difference and the time of fall shows that if the temperature is constant, then

$$(1-x)t_1=(1-2x)t_2$$

or

$$x=\frac{t_2-t_1}{2t_2-t_1}$$

If t is the time of fall which would be obtained if the full hydrostatic pressure were operative, then

$$t=t_1(1-x)$$

Except for a small correction for slip,* which can be applied independently, t is proportional to the viscosity of the gas in the apparatus. Direct comparisons of viscosities thus amounted simply to the computation of the ratio of such corrected times.

Since the capillary effect may change slowly with time, the following procedure was adopted throughout in order to allow for its effective elimination. Eight observations of the time of fall were made with the pellet intact, and eight with the pellet in two segments. The order of observation was such that the second group lay between the first four and the last four of group one. The subjoined table gives a typical case and also serves to indicate the consistency of the observations. In this way any slow change in the capillary effect would certainly be eliminated.

TABLE I.

Temp. (deg.C.)	Time of fall in seconds.			
	Whole pellet.		Two segments.	
	A to B.	B to A.	A to B.	B to A.
15·9	160·3	161·0	...	...
15·9	160·5	161·0	...	...
15·95	...	...	167·1	168·4
15·93	...	...	167·6	168·5
15·9	...	...	167·6	168·2
15·9	...	...	167·8	168·5
15·9	160·5	161·3	...	...
15·93	160·5	161·7	...	...

Therefore t_1=160·85 secs., t_2=167·96 secs., giving for the capillary correction

$$x=0\cdot0406$$

whence the corrected time of fall is

$$t=154\cdot32 \text{ secs. at } 15\cdot9°C.$$

* A. O. Rankine, Proc. Roy. Soc., A. Vol. LXXXIII., p. 517 (1910).

The apparatus was cleaned with distilled water and absolute alcohol, and finally dried by being exhausted and connected to a bulb surrounded by liquid air. In order to obtain a smooth motion of the mercury pellet in the fall tube, it is necessary that the gases which are introduced into the viscometer shall be quite dry, and special precautions were taken to desiccate all the gases used. It is worthy of note that although the apparatus remained in use for more than five weeks, the motion of the pellet was just as good at the end of the period as at the beginning.

Preparation and Purification of the Gases.

(*a*) *Air.*—The air which was used as a standard, in order to be able to calculate the absolute viscosity of the gases, was dried by being passed over phosphorus pentoxide, and then through a spiral maintained at −80°C. by means of a mixture of solid carbon dioxide and alcohol, before being introduced into the viscometer. This procedure removes the water vapour very effectively.

(*b*) *Carbon Dioxide.*—The sample of carbon dioxide which was used was prepared by the action of dilute sulphuric acid on pure anhydrous sodium carbonate. Possible traces of sulphur dioxide and water were respectively removed by passing the gas through a dilute solution of potassium permanganate, and then through concentrated sulphuric acid. The gas was then solidified in a U-tube surrounded by liquid air, and samples were finally collected over mercury. Before introduction into the viscometer the gas was again solidified by means of liquid air, and any permanent gases—the only likely impurities which may have remained in any quantity—were pumped off by means of a Toepler pump. When the liquid air was removed the carbon dioxide was collected in the pump, and then introduced directly into the evacuated viscometer. All the CO_2 had not evaporated before the sample was introduced into the apparatus ; thus any impurity of higher boiling point would be retained in the condenser.

(*c*) *Nitrous Oxide.*—The specimen of nitrous oxide was obtained from a cylinder of the gas of commercial quality. The gas was allowed to bubble slowly through a solution of caustic potash and another of ferrous sulphate, to remove traces of carbon dioxide and chlorine, and nitric oxide respectively, and then partially dried by being passed through a U-tube surrounded by a freezing mixture, and finally solidified in a U-tube maintained at liquid air temperature. The gas was collected over mercury and introduced into the viscometer with the same precautions as were observed in the case of carbon dioxide.

(*d*) *Nitrogen.*—Recently Waran * has described a method of making pure nitrogen in small quantities, which when examined spectroscopically is free from hydrogen. Attempts to use the same apparatus to obtain larger quantities of nitrogen resulted in the production of ammonia, as an impurity, in such great quantity that the process of purification was very laborious. The following modification of Waran's apparatus was therefore used. The portion of the apparatus *A B C D* was completely filled with dilute ammonia, and dilute bromine solution was placed in the thistle funnel above *B*. The delivery tube *E* dipped under an inverted tube *F*, and both were filled with recently boiled distilled water. The inverted tube *F* was closed by means of a tap K, and was connected to an exhausted glass spiral immersed in liquid air. When *K* was opened the nitrogen passed into the spiral, and most of the water

* Waran, Phil. Mag., Aug. (1921).

and possible traces of ammonia which might still be present would be retained in the spiral tube. The sample of gas was again purified by being passed slowly through a spiral tube immersed in liquid air, before being introduced into the viscometer.

(*e*) *Carbon Monoxide.*—This gas was also generated in the above apparatus, concentrated sulphuric acid replacing the ammonia and formic acid the bromine solution. It was observed that sulphuric acid attacked the tap grease liberating carbon, so that traces of sulphur dioxide and carbon dioxide may at first be present

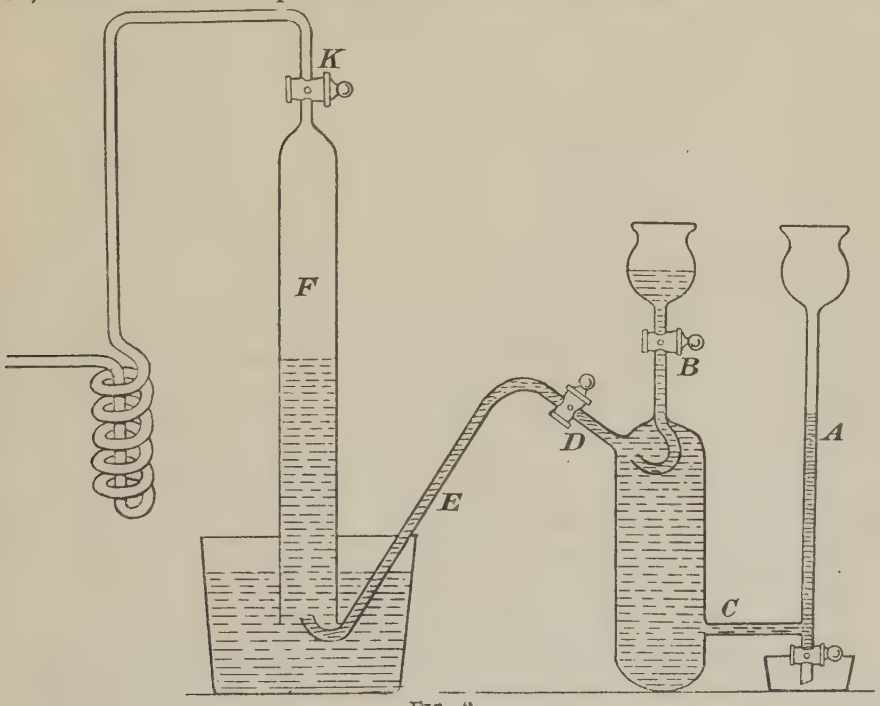

FIG. 2.

in the carbon monoxide. These gases would certainly be removed by means of the liquid air. The gas was collected and purified as in the case of the nitrogen.

RESULTS.

In Tables II.—V. the letters in column 1 indicate the order in which the observations were made.

TABLE II.—*Carbon Dioxide.*

Temp. (deg. C.)	Time of fall (seconds).		Capillary correction (x).	Corrected time (t).	Time at	
	Whole pellet.	Two segments.			15·0° C.	100·0° C.
(*a*) 16·2	133·99	139·75	0·0396	128·68	128·20	...
(*d*) 18·2	134·56	140·16	0·0385	129·38	128·10	...
				Mean	128·15	...
(*b*) 100·2	168·58	172·99	0·0249	164·38	...	164·30
(*c*) 100·0	167·90	171·76	0·0219	164·22	...	164·20
				Mean	...	164·25

TABLE III.—*Nitrous Oxide.*

Temp. (deg. C.)	Time of fall (seconds).		Capillary correction (x).	Corrected time (t).	Time at	
	Whole pellet.	Two segments.			15·0° C.	100·0° C.
(*a*) 16·3	132·83	137·23	0·0311	128·70	128·18	...
(*d*) 17·9	132·74	136·61	0·0275	129·09	127·93	...
				Mean	128·06	...
(*b*) 99·6	167·29	171·14	0·0220	163·61	...	163·77
(*c*) 99·6	167·06	170·56	0·0201	163·70	...	163·86
				Mean	...	163·82

TABLE IV.—*Nitrogen.*

Temp. (deg. C.)	Time of fall (seconds).		Capillary correction (x).	Corrected time (t).	Time at	
	Whole pellet.	Two segments.			15·0° C.	100·0° C.
(*a*) 15·9	160·85	167·96	0·0406	154·32	153·96	...
(*c*) 15·4	160·41	167·41	0·0401	153·97	153·79	...
(*d*) 15·5	160·74	167·96	0·0412	154·12	153·92	...
(*e*) 16·1	160·73	167·75	0·0401	154·28	153·84	...
				Mean	153·88	...
(*b*) 100·0	193·68	200·21	0·0316	187·56	...	187·56
(*f*) 99·9	193·45	199·75	0·0306	187·53	...	187·59
				Mean	...	187·58

TABLE V.—*Carbon Monoxide.*

Temp. (deg. C.)	Time of fall (seconds).		Capillary correction (x).	Corrected time (t).	Time at	
	Whole pellet.	Two segments.			15·0° C.	100·0° C.
(*a*) 18·2	161·90	169·28	0·0418	155·13	153·85	...
(*b*) 18·5	162·28	167·73	0·0420	155·46	154·06	...
(*d*) 17·4	161·39	168·59	0·0407	154·82	153·86	...
				Mean	153·92	...
(*c*) 99·7	193·39	199·49	0·0297	187·65	...	187·77
(*e*) 99·6	193·14	199·23	0·0296	187·42	...	187·58
				Mean	...	187·68

Throughout the foregoing tables the figures in the last two columns have been obtained by applying the small corrections necessary to obtain values at the two standard temperatures indicated, for purposes of comparison. The basis of the corrections is the observed temperature rate of change of the time of fall.

EXAMINATION OF RESULTS.

Taking first of all the gases CO_2 and N_2O, the mean values of the corrected times of fall are shown in Table VI.

TABLE VI.—*Carbon Dioxide and Nitrous Oxide.*

Temp. (deg. C.)	Gas.	Time of fall (seconds).	Ratio.
15·0	CO_2	128·15	1·001
	N_2O	128·06	
100·0	CO_2	164·25	1·003
	N_2O	163·82	

Here the time of fall is strictly proportional to the viscosity of the gas.* A careful examination of the detailed results shows that the experimental error is not likely to be more than 0·2 per cent. At 15·0°C. the ratio of the viscosities is closer to unity than this ; and even at 100·0°C. the excess of 0·003 is not sufficient to make it safe to claim that a difference of viscosity has been detected. In any case the remarkable result emerges that both at 15·0°C. and 100·0°C. the viscosities of carbon dioxide and nitrous oxide are equal to within 0·3 per cent.

In Table VII. the data for carbon monoxide and nitrogen are given.

TABLE VII.—*Carbon Monoxide and Nitrogen.*

Temp. (deg. C.)	Gas.	Time of fall (seconds).	Ratio.
15·0	CO	153·92	1·000
	N_2	153·88	
100·0	CO	187·68	1·001
	N_2	187·58	

In this case both ratios are nearer to unity than 0·2 per cent. ; so that we can say that the measurements indicate an even more precise equality of viscous behaviour than in the previous case.

Thus, these experiments prove, as closely as the method allows, that the two pairs of gases in question have, respectively, identical viscous properties at both atmospheric and steam temperatures.

In these circumstances the proper course is to disregard the small differences in the figures and to take the mean value of the times of fall as being a measure of the common viscosity. This has been done for the purpose of calculating Sutherland's constant, and the absolute viscosity by comparison with air.

SUTHERLAND'S CONSTANTS.

These have been calculated directly from the ratios of the viscosities at the two temperatures, on the assumption that Sutherland's law is valid.

TABLE VIII.

Gas.	$\frac{t_{100}}{t_{15}} = \frac{\eta_{100}}{\eta_{15}}$	Sutherland's Constant. (C).
CO_2 and N_2O	1·280	274
CO and N_2	1·219	118

Here it should be noted that the accuracy of Sutherland's constant is not great, being of the order of 5 per cent. For so small a temperature range the constant is very sensitive to variations of the viscosity ratio, in which there is a possible error of 0·3 per cent.

ABSOLUTE VALUES OF THE VISCOSITIES.

These have been obtained simply by comparing the times of fall in the gases with those of the same mercury pellet in air. For this purpose many observations

* In general this would not be true. But for gases, having as these have, the same density and the same approximate viscosity, their slipping coefficients are equal, and therefore do not affect the above proportions.

were made using air in the viscometer. Small corrections have to be applied on account of the different slipping coefficients, in the manner already indicated.*

The results are as follows :—

For CO_2 and N_2O,

$$\frac{\eta_{15}}{\eta_{15(air)}} = 0{\cdot}8021$$

which when corrected for slip becomes

$$0{\cdot}8011$$

For CO and N_2,

$$\frac{\eta_{15}}{\eta_{15(air)}} = 0{\cdot}9654$$

Here correction for slip is negligible.

Taking $1{\cdot}799 \times 10^{-4}$ C.G.S. units as the absolute viscosity of air at 15·0°C., the following table shows the absolute viscosities for the gases under investigation. The values at 0·0°C. have been obtained by extrapolation.

TABLE IX.—*Summary of Results.*

Gas.	Viscosity in C.G.S. units $\times 10^{-4}$ at			C.
	15·0°C.	100·0°C.	0·0°C.	
CO_2 and N_2O	1·441	1·845	1·366	274
CO and N_2	1·737	2·118	1·665	118

DISCUSSION OF RESULTS.

The value of the viscosity at 0·0°C. for CO_2 and N_2O obtained in this research is $1{\cdot}366 \times 10^{-4}$ C.G.S. units. Probably the most reliable previous measurements for these gases are those due to Vogel.† His results are : for CO_2, $\eta_0 = 1{\cdot}380 \times 10^{-4}$ C.G.S. units, and for N_2O, $\eta_0 = 1{\cdot}362 \times 10^{-4}$ C.G.S. units.

The absolute viscosity of nitrogen has recently been determined by Prof. Kia-Lok Yen‡ in Millikan's laboratory, and he found $\eta_{23} = 1{\cdot}765 \times 10^{-4}$ C.G.S. units, whereas the present result obtained by interpolation is $\eta_{23} = 1{\cdot}773 \times 10^{-4}$ C.G.S. units. Vogel gives $\eta_0 = 1{\cdot}678 \times 10^{-4}$ C.G.S. units, and for carbon monoxide $\eta_0 = 1{\cdot}672 \times 10^{-4}$ C.G.S. units. The common value shown in the above table is $1{\cdot}665 \times 10^{-4}$ C.G.S. units.

Previous work on the variation of viscosity with temperature does not appear to have been sufficiently systematic to render of value comparisons with the present results.

CALCULATION OF MOLECULAR DIMENSIONS.

We can now proceed to calculate the mean collision area ($\bar{A}$) for the molecules, CO_2 and N_2O, and CO and N_2, by applying Chapman's formula in the manner previously indicated,§ and we find

$$\bar{A} = 0{\cdot}834 \times 10^{-15} \text{ sq. cm. for } CO_2 \text{ and } N_2O,$$

and

$$\bar{A} = 0{\cdot}767 \times 10^{-15} \text{ sq. cm. for CO and } N_2.$$

* A. O. Rankine, Proc. Roy. Soc., A. Vol. LXXXIII., p. 517 (1910).
† H. Vogel, Berlin Diss. (1914).
‡ Kia-Lok Yen, Phil. Mag., Vol. XXXVIII., p. 593 (1919).
§ A. O. Rankine and C. J. Smith, Phil. Mag., Vol. XLII., p. 612 (1921).

Conclusion.

The fact that CO_2 and N_2O (or CO and N_2) have molecular dimensions which are identical within the accuracy of measurement is consistent with Langmuir's view, to which reference has been made ; namely, that the outer electron arrangements of these molecules are the same. The present experiments indicate that this is the case with considerably greater precision than could have been deduced from previous data. It should be remembered, however, that independently of any particular theory of molecular structure, it is unlikely that CO_2 or N_2O (or CO and N_2) would differ much in those physical properties—viscous behaviour being one—which depend on the total mass of the molecule and the number of electrons external to the nuclei. For CO_2 and N_2O have equal molecular weights, viz., 44, and equal numbers of extra-nuclear electrons, viz., 16 ; and CO and N_2 both have a molecular weight of 28 and extra-nuclear electrons 10 in number. The equalities of apparent molecular size which have been indicated do not of themselves prove any special electronic arrangement ; but merely suggest that, whatever the arrangements are, they are nearly identical.

To detect appreciable differences we should almost certainly have to examine properties dependent on the distribution of atomic nuclei in the molecules. For example we may expect differences in the molecular moments of inertia. The moment of inertia is almost entirely attributable to the masses of the nuclei of the constituent atoms, the electrons making an insignificant contribution to the mass. Even if the electrons in CO_2 and N_2O (or CO and N_2) are distributed in an identical manner, the molecular moments of inertia will be unequal on account of the differing masses of the atoms in the molecule. It seems probable that the spectroscopic method, applied by Imes* and others to the gaseous hydrogen halides, would lead to the discovery of such differences of rotational inertia. This would involve the measurement of the far infra-red absorption spectra for the gases in question, and the treatment of the results in terms of the quantum theory.

In conclusion, I wish to thank Professor Rankine for his unfailing help and advice while these experiments have been made, and also the Government Grant Committee of the Royal Society for a grant which enabled the research to be undertaken.

DISCUSSION.

Prof. A. O. Rankine : The author, who has worked with praiseworthy perseverance, has succeeded in establishing beyond dispute a close identity in viscous properties between the two pairs of gases investigated. The *ratios* of the viscosities have been measured with an extremely high accuracy, since they are the ratios of times which are practically equal. For the *absolute* viscosities, however, less accuracy must be claimed, as their measurement depends on the calibration of a stop-watch. It is remarkable that gases which resemble one another so closely in their physical properties should differ so widely in other—*e.g.*, their physiological properties. It seems clear that chemical and physiological properties are determined by the distribution of the nuclei, and are independent of the outer electron system. A further curious fact has been brought to light—viz., that while the surface-tension effects of mercury are the same when in contact with various gases which differ widely in other respects, the behaviour of a mercury pellet in presence of CO_2 is very different from that in presence of N_2O. Dr. Percy Phillips, in an unpublished investigation, attempted to obtain the variation of the viscosity of N_2O in the neighbourhood of the critical state, but failed because the angle of contact of the mercury with the glass became 180 deg., so that the gas slipped past the pellet, whereas no such difficulty had arisen in the case of CO_2 near the critical state.

* Imes, Astrophysic. Journ., Vol. L., p. 251 (1919).

Dr. D. Owen congratulated the author on his work, the interest of which arose from the connection which the viscosity of a gas bore with the size and shape of the molecule. The author remarks that all theories of atomic structure which identify the electron system of the molecule of N_2 with that of CO can equally claim the support of his results. But since the Lewis-Langmuir theory does lead to a close equality in structure in the case of the gases in question, it seemed fair to place the agreement found in the present Paper to the credit of that theory. In regarding theories of the atom and molecule one wondered whether there were certain properties only explicable on a statical theory, others requiring a dynamical theory; one was reminded of the apparent antinomy in the province of radiation between the undulatory and the quantum theories. The accuracy obtained by the device of splitting the mercury pellet—viz., ·2 per cent.—is remarkably good. Even in very carefully cleaned tubes it appears that the correction in the case of a single pellet may amount to 4 per cent., and it is subject of congratulation that by the device of splitting the pellet this has been reduced to $\frac{1}{5}$ per cent. He would like, in conclusion, to ask the author whether he could give a simple physical interpretation of Sutherland's Constant—which has the dimensions of a temperature, and which appears so constantly in relation to subjects like that of the present Paper.

Dr. F. Ll. Hopwood remarked that the suggestion hardly commended itself that physiological properties depended on the nuclei, but rather on the *configuration* of the constituent atoms or *shape* of the molecule (as indicated by the experiments of Prof. Hamburger). The author's results gave a measure of the mean collision *area* of the molecule, and did not necessarily give any information concerning its *configuration*.

Mr. R. S. Whipple suggested that a photographic method might be of advantage in measuring the time of fall of the pellets.

Dr. H. P. Waran: I would suggest that the difficulty which the author experienced in obtaining nitrogen free from ammonia was due to his using too strong a solution of the latter. My original apparatus can be modified by providing a side tube near the tube *C*, Fig. 2, and maintaining a steady flow of the reacting liquids into and out of the apparatus. Very dilute ammonia can then be used, and the trace of this substance in the resulting nitrogen will be so small that it can be absorbed along with the water vapour by the acid phosphorus pentoxide in the bulb between *K* and *D*, the parts *E*, *F* being dispensed with. Nitrogen thus obtained is spectroscopically pure, and any quantity can be generated.

Dr. J. S. G. Thomas: The values of the viscosities of gases are of such importance to gas technologists that results over a wider range of temperature would be extremely valuable. The work of Stanton and Pannell on flow of gases, and the formula deduced from their results by Lees, constitute a very valuable contribution from pure science to technology, and the value of this contribution would be still greater were values of the viscosities of gases, and more especially of mixtures of gases, available at temperatures up to, say, 600°C. Little seems to have been done in the matter of determining the relation of the viscosity of a gaseous mixture to the viscosities of its constituents. The best that can be done for purposes of technology is to calculate the viscosity of the mixture from its composition, the viscosities of the several constituents at the appropriate temperature being calculated from the values available, by the application of Sutherland's formula. The procedure is a very doubtful one, and it is not surprising to find that the pressure drop actually found in a pipe system through which, say, coal gas at 300°C. is flowing, is very different from that determined by calculation. The accurate determination of viscosities of pure gases and mixtures at higher temperatures would be of value not only from the point of view of the determination of atomic structure, but also from the point of view of the practical application of the results.

The Author, replying to Dr. Owen: The results which have been obtained for CO_2 and N_2O (or CO and N_2) show that the electronic configurations of these molecules, as nearly as the precision of the method enables us to say, give rise to *mean* collision areas which are identical. Langmuir places the extra-nuclear electrons in the same positions for each pair of these gases, and hence the results support his theory, but they also support any theory of molecular structure which places these electrons in identical positions. He also says, however, that the molecules of CO_2 and N_2O would have the same shape as three contiguous neon atoms in a line. Professor Rankine has shown that this is the case. It is in this latter respect that the results of the present research lend definite support to the Lewis-Langmuir theory.

In regard to Sutherland's constant C, which appears in the formula

$$\eta = k \frac{T^{\frac{1}{2}}}{1+C/T}$$

(k=const. T=absol. temp.)

C, as it stands in the formula, is clearly a temperature, and it is proportional to the work done in separating two molecules originally in contact to an infinite distance apart.

It may be of interest to quote a remark by Dr. Oxley which will be found in Nature, July 21, 1921, p. 652: "On the cubical atom theory as developed by Lewis and Langmuir it appears that the molecules of CO_2 and N_2O have almost identical electron configurations. A. O. Rankine has shown from viscosity data that each electron system is equivalent to that of three adjoining neon atoms in line. The writer is not aware that attention has been directed to the fact that the specific susceptibilities of gaseous CO_2 and N_2O are $-0{\cdot}423\times10^{-6}$ and $-0{\cdot}429\times10^{-6}$ (T. Soné, Science Reps. Tohoku, Vol. VIII., p. 162, 1919, and Proc. Phys. and Math. Soc., Japan, Vol. II., p. 84, 1920), and their molecular susceptibilities $-18{\cdot}6\times10^{-6}$ and $-18{\cdot}8\times10^{-6}$ respectively.

"The electron systems of the two molecules are apparently identical, but the net positive charges on the nuclei are 8—6—8 for CO_2 and 7—8—7 for N_2O. If, therefore, atomic nuclei possess rotations and are a source of magnetic moment, it must be assumed that the re-distribution of the positive charges in the manner indicated involves no change of angular momentum."

DEMONSTRATION of the Optical Sonometer. By F. Twyman, *F.Inst.P.* (*A. Hilger, Ltd.*).

The apparatus is designed to record the pressure variation caused by sound waves. The sound wave is received by the horn A attached to the diaphragm box B (*see* Fig. 1). In the box is a diaphragm C, consisting of a celluloid film a fraction of a wave-length in thickness, a small portion of the inner face of which is platinized, silvered or gilt.* Light is directed from the light source D by a condenser E through the slit F, and brought to a focus on the diaphragm C. Lenses G and H form an image of the slit on the photographic paper (or film) on the drum J. The lens H being cylindrical, with its axis parallel to the drum, the beam of light is brought to an intense point image, so that when the drum rotates a record of the deflection of the diaphragm is made. The apparatus is arranged to take a 30 or 100 candle-power Pointolite lamp.

Liberal adjustment is provided, so that the spot of light may be focussed on the the drum at any distance from 4 to 20 inches, according to the amplitude of vibration to be dealt with.

The recording drum is arranged to take celluloid or paper film $1\frac{3}{8}$ inches wide. For general use, the paper film will be found sufficiently sensitive, but in cases when the amplitude is large and waves of high frequency, standard cinematograph film will be found advantageous on account of its superior speed and orthochromatic quality.† On account of its orthochromatic quality, cinema film is more suitable for use with the gilt diaphragm.

The drum is enclosed in a light tight box, and provided with an automatic shutter, arranged to expose from one-sixth to one complete revolution of the drum, as desired; thus, from 1 to 6 records can be made on one film according to the length it is desired to expose. The drum takes 37 inches of film.

The drum is driven by a spring motor provided with a speed indicator. The speeds thus shown are, however, only to be taken as a rough guide, and when accurate timing is required a time marker should be used.

Time Marker.

This consists of an electrically-driven tuning-fork (placed at some distance from the apparatus in order to guard against interference with the sound to be recorded), such that a small mirror on the fork reflects light taken from the light source of the recording apparatus, back to the drum, where it is focussed near the edge of the film; both sets of variations are photographed together, and thus an accurate time reference is secured.

The apparatus can also be supplied with the axis of the diaphragm in a horizontal plane.

Examples of Records.

Three records showing the sound waves of a mallet, human voice and whistling are reproduced (Figs. 2, 3, 4).

One must not, of course, accept these curves as representing the time variation of air pressure without investigation.

* The silver gives better reflection while untarnished, but the gilt is more durable.

† Kodak Cine ortho film is about 300 H & D. Paper film about 75 H & D.

The free period of vibration of the membranes of the size here shown in use appears to be approximately 200 per second. On singing up the scale one finds, either with or without the use of the horn a big increase of response at about that

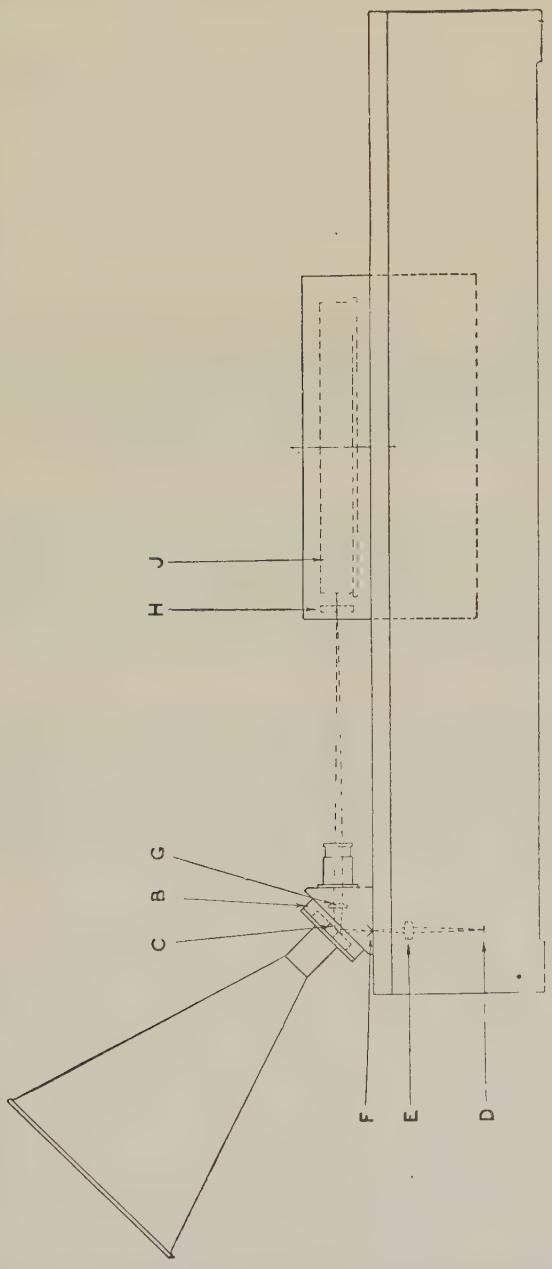

FIG. 1.—OPTICAL SONOMETER.

pitch; the response is so great in this immediate neighbourhood as to obliterate by contrast the characteristic overtones of the impressed vibration. But if one

keeps away from the immediate neighbourhood of this fundamental note, one feels reasonably sure that a good qualitative record is given of the pressure variations, and one arrives at conviction on this point from the records of a number of different sounds.

If it should be desired to know variations of pressure quantitatively, the effect of the constants of the film and their reactions must be calculated. It may be of interest to quote ways in which the constants of a film may be in practice determined.

1. *Mass of the Film.*

If on an interferometer the curvature of the film be determined when it is horizontal by observation from above and from below, and if, in addition, an observation be made with the film vertical, the mass of the film may be determined as a function of its diameter and tension.

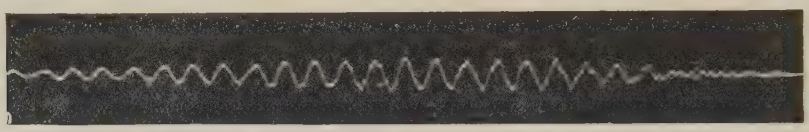

FIG. 2.—LEATHER COVERED MALLET ON WOOD. FREQUENCY ABOUT 250 PER SEC.

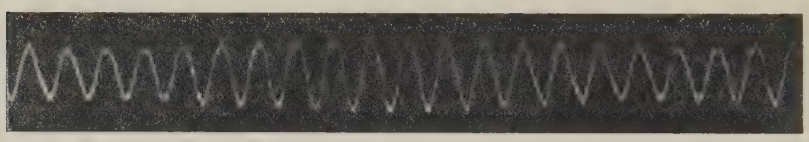

FIG. 3.—SINGING. FREQUENCY ABOUT 200 PER SEC.

FIG. 4.—WHISTLING. FREQUENCY ABOUT 1,300 PER SEC.

2. *Tension of the Film.*

This can be determined in two ways: By application of air pressure, or by observing the additional deflection obtained with a horizontal film on placing a small ring of known weight upon it in a concentric position, the observations being made, as in the former case, on an interferometer.

One would hope that analysis would reveal that variation of the tension with deflection of the membrane in the region of operative pressures could be neglected.

3. *Energy Dissipation of the Film in Vibrating.*

The information obtained from (1) and (2) together, with the physical properties of the film, are sufficient to enable the energy dissipation to be calculated.

This must be known if the pressure variation in the region of the sympathetic vibrations are required, since it is the dissipation of energy which determines the amplitude.

XXIV. *A New Apparatus for the Measurement of the Polarisation-Capacities of Platinum Plates in Sulphuric Acid. By* A. Griffiths, *D.Sc., and* W. T. Heys, *B.Sc., Birkbeck College, London.*

Received February 23, 1922.

ABSTRACT.

A new and simple apparatus for the measurement of the capacities of electrolytic cells has been developed, which allows corrections to be made for the leakage and self-depolarisation of the cell. It gives results consistent to one or two per cent., and of the same order of magnitude as those obtained by other observers.

(1). Introduction.

The authors are engaged in a study* of the variation of the polarisation-capacity of an electrolytic cell, consisting of platinum plates in sulphuric acid, when subjected to an intermittent current, and they have developed an apparatus containing some original features which are described below. In order to correct for the rapid leakage or self-depolarisation which apparently constitutes nearly all the difficulty in measurements of this kind, they employ a falling-plate method for making connections of short, variable, and calculable periods.

(2). Method.

In Fig. 1 E denotes a 2, 4, or 6-volt battery, x the electrolytic cell, C the charging condenser (1 microfarad capacity), K the auxiliary condenser (4 microfarads capacity) and G the ballistic galvanometer.

The dots numbered 1 to 6 represent brushes which are connected to the component parts of the apparatus, as suggested in Fig. 1. Fixed on a heavy insulating carrier are two brass plates P and Q (Fig. 2), whose shapes and relative positions are so chosen that as the plates fall between vertical guides past the brushes. The following momentary contacts are made :—

The battery E is connected to the condenser C, charging it. The condenser C is connected in parallel with the electrolytic cell x and the auxiliary condenser K, and thus shares its charge with them. The condenser K is discharged through the galvanometer G.

Immediately after the breaking of the contacts between C, x and K, both C and K have acquired voltages equal to the polarisation back E.M.F. of the electrolytic cell x. As the condenser C is affected by the residual charge, not it but the condenser K is discharged through the galvanometer G to determine the value of the back E.M.F. of the electrolytic cell.

Regarding the cell as a pure capacity of 20 microfarad in series with an electrolytic resistance of 1^{ω}, and taking the resistances of the connecting wires and brushes in series with the condensers C and K as $1/100^{\omega}$ each, it can be shown that the cell will receive all of its final charge except one-tenth per cent. in less than 10^{-5} secs. The cell x is, of course, connected to the condensers C and K for a much longer time than this, so that the effect of the self-depolarisation of the cell may not be inappreciable. Examination of Fig. 2 will show that the brush 4 attached to the electrolytic cell x may by a lateral displacement cross the plate P by any of the paths α, β or γ, and thus that the time during which the cell is in contact with the condensers C

* Phil. Mag., March, 1895, p. 232 ; Proc. Phys. Soc., Vol. XXIX., Part IV., June 15th (1917).

and K may easily be varied by intervals calculable in terms of the (constant) height of fall of the carrier and the length of the path of the brush. By plotting the values of the back E.M.F. of the cell against the time during which it has been free to depolarise, we may extrapolate and thus obtain the initial value of the back E.M.F. of the cell.

(3). RESULTS.

(i.) *Effect of Leakage and Calculation of Results.*—The back E.M.F. of the electrolytic cell was measured at intervals of 0·002, 0·004, and 0·006 seconds after the

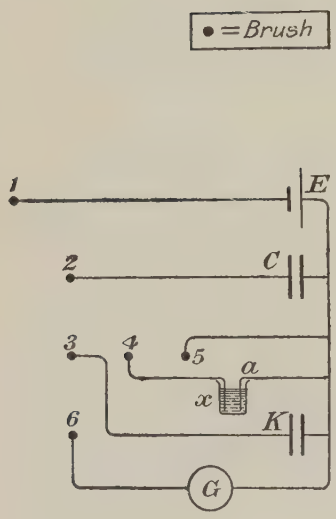

FIG. 1.

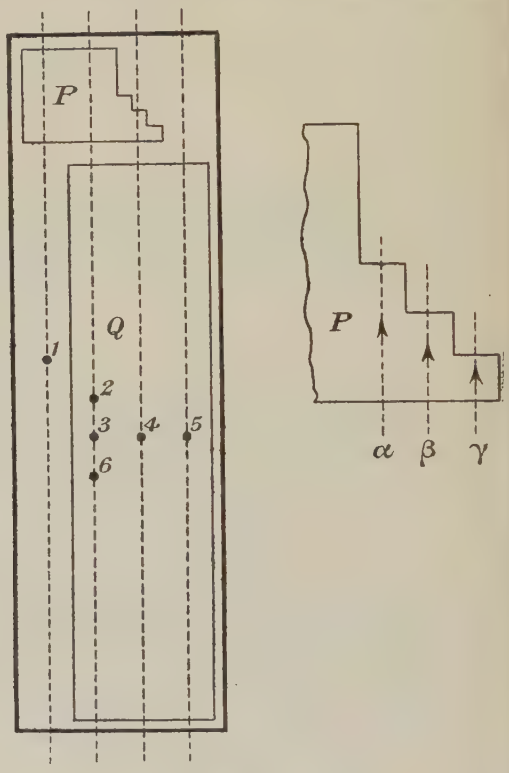

FIG. 2.

connection of the condenser C with the cell x and the condenser K. Some typical results with two bright platinum electrodes 1 cm. × 1·5 cm. immersed in dilute sulphuric acid (1 part conc. acid to 12 parts distilled water) are given below :—

Time of self-depolarisation in seconds (approx.).	Back electromotive-force in volts.	
	October 14.	November 17.
0·002	0·1073	0·1095
0·004	0·1057	0·1063
0·006	0·1036	0·1047

Extrapolation by means of the graph (Fig. 3) gives the initial back E.M.F.'s as 0·1095 and 0·112 volts, so that the capacities of the cell on the two dates were 13·3 and 12·9 microfarad respectively.

(ii.) *Variation of the Capacity of Electrolytic Cells with Time and with Different Specimens of Electrodes.*—In every case the electrodes were rectangular—1 cm. by 1·5 cm.

Date.	Capacity of cell in microfarads.
August 15th, 1921	46·3
September 29th	17·2
October 14th	13·3
November 17th	12·9

Electrodes.	Capacity in microfarads.
A and *B*	12·9
C and *D*	8·7
P and *Q*	5·0

(iii.) *Variation of the Back E.M.F. of an Electrolytic Cell with the Charge to which it is Subjected.*—The back E.M.F. of an electrolytic cell produced by the passage through the cell of a definite quantity of electricity is not proportional to

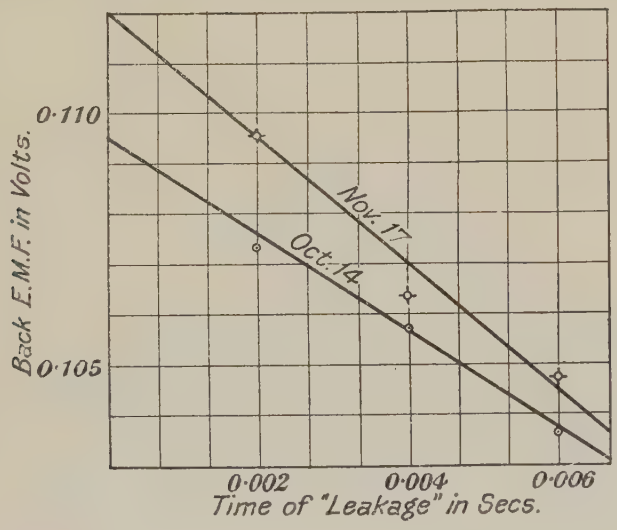

FIG. 3.

the charge passed, but increases more slowly than the latter. This indicates an increase in the capacity of the cell.

Charge passed through the cell in micro-coulombs.	Back electromotive-force of the cell in volts.
1·56	0·091
3·17	0·167
6·74	0·232
12·34	0·332
21·98	0·404
29·82	0·437

It may be added that the back E.M.F. developed depends to a slight extent upon the direction in which the cell is charged.

(iv.) *Measurement of the Capacity of the Cell for Small Charges.*—On repeating any measurement slight discrepancies are often found, seldom amounting to 2 per cent. It was found that if the battery of accumulators be removed from the circuit and the plate dropped as usual, a small deflection was still obtained. The following is a

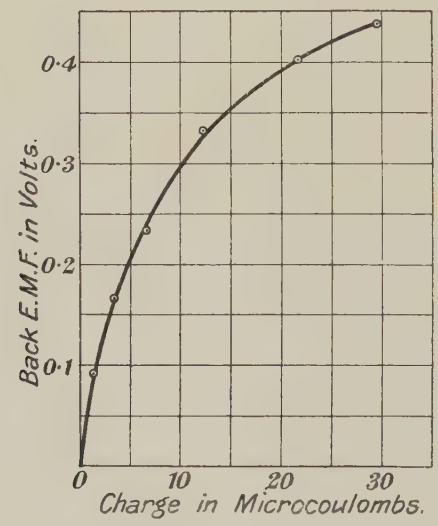

FIG. 4.

series of typical readings (in cms.): +0·25, +0·65, +0·25, −0·25, +0·25, −0·3, −0·15, +0·4, +0·35, +0·35. To make some allowance for the results of these "dummy" experiments, a dummy experiment was performed before and after each ordinary experiment, and the mean of the deflections in the dummy experiments subtracted from the deflection obtained in the ordinary experiment. As an example the following series of readings is appended :—

Order of experiment.	Deflection in cm.	Mean of deflections in 1, 3 and 5.	Corrected deflections.
1. (Dummy)	+0·15	...	...
2.	+7·0	...	+6·83
3. (Dummy)	+0·2	+0·17	...
4.	+7·0	...	+6·83
5. (Dummy)	+0·15	...	...

The following table gives the results of a series of experiments in which the above correction was applied :—

Order of experiment.	Anode of cell.	Voltage of charging condenser.	Galvanometer deflection in cm. Time of leakage in secs.			Initial value of back E.M.F. in volts.	Charges passed through cell in micro-coulombs.
			0·002	0·004	0·006		
1	C	2	6·83	6·78	6·56	0·140	0·960
9	C	2	6·95	6·85	6·82	0·140	0·960
3	C	4	12·80*	12·95	12·88	...	...
7	C	4	12·88	12·81	12·80	0·259	1·977
5	C	6	18·27	18·19	18·16	0·367	3·023
2	D	2	7·09	6·98	6·90	0·144	0·950
10	D	2	7·02	7·02	6·93	0·142	0·954
4	D	4	13·84	13·78	13·73	0·278	1·966
8	D	4	13·75	13·69	13·65	0·276	1·930
6	D	6	19·98	19·93	19·90	0·403	2·926

Capacity of charging condenser $C=\frac{2}{3}$ microfarad.

Capacity of auxiliary condenser $K=2$ microfarads.

Electrodes C and D of bright platinum, 1×1 sq. cm., in dilute sulphuric acid.

The graph (Fig. 5) shows that there is an approximately linear relationship

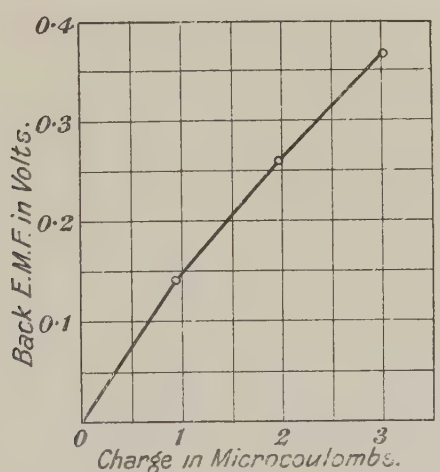

FIG. 5.

between the charge passed and the back E.M.F. developed. The capacity of the cell for small charges=6·85 microfarads, *i.e.*, 0·068 microfarads per sq. mm. for each electrode.

In conclusion the authors desire to express their indebtedness to Messrs. H. G. Bell and G. Temple for help and suggestions.

* It will be noted that out of all the results only one has to be rejected.

DISCUSSION.

Dr. D. Owen: I should like to ask the authors whether they have worked with platinum-black electrodes, and if so with what results. The authors, like others who have worked at this subject, use the term "Polarisation Capacity." The value of this, for plates of area of 1 cm.^{2}, is calculated to be of the order of 10 microfarads. The cell is thus regarded as electrically equivalent to a resistance (of the order of 1 ohm in the present experiments) in series with a leaky condenser. Judging from the curves of Fig. 3, the resistance of the leak is several thousands of ohms. Now, if such a system be subjected to an alternating voltage it should show marked signs of the presence of this electrostatic capacity. It should have a considerable reactance, far greater than the ohmic resistance of the acid of the cell, which should reduce the value of the current and at the same time cause this to lead the voltage considerably in regard to phase. Accurate measurement of electrolytic resistance by alternating currents should thus be rendered difficult or impossible—which is quite contrary to what is known to be the case. It must be inferred that the interpretation of the phenomena of electrolytic polarisation at electrodes in terms of an assumed electrostatic capacity is not in general justifiable; though doubtless with electrodes of certain metals, such as aluminium, in appropriate electrolytes, such capacity may be present.

Dr. A. Russell: The Society is much indebted to the authors for bringing this subject before it, as the conception of "polarisation capacity" is not a generally familiar one. Experience on A. C. circuits with plates immersed in soda, for instance, shows that such plates act in some ways like a condenser of very variable capacity.

Dr. Griffiths: I am not sure what Dr. Owen means by "leakage-resistance." I think that changes take place on the surface of the platinum electrode, and also in the liquid in the neighbourhood of the electrode. The surface tends to come back to its original condition; and through diffusion the liquid tends to revert to its original uniformity. Until more is known of the subject it does not seem unwise to follow the example of others in using the term "polarisation-capacity." As a matter of fact, appreciable phase differences have been obtained by some workers. The term "leakage-resistance" does not occur in the Paper, and I have avoided it in presenting the latter. I have tried to determine the capacity of platinum-black, but have not succeeded with my present apparatus. The rate of depolarisation is so great that extrapolation is not permissible with the experimental results obtained. Success might be obtained if the time of connection were reduced considerably, or, as the rate of depolarisation will diminish with diminished electromotive forces, if feebler charges were given to the electrolytic cell, and if my present galvanometer were replaced by one much more sensitive. Success might also be attained by using an auxiliary condenser of great capacity and with smaller charging E.M.F.s.

DEMONSTRATION of Some Electrical Properties of Neon-filled Lamps. By Messrs. S. O. PEARSON, *B.Sc., and* H. ST. G. ANSON.

THE Demonstration exhibited the behaviour of "Osglim" lamps (made by the General Electric Company) when shunted by a condenser and connected in series with a high resistance to a D.C. voltage supply. In these circumstances an intermittent current passes through the lamp, as indicated by the intermittency of the orange glow on the cathode (formed in this case as a letter of the alphabet of conconsiderable area, the anode being much smaller); this fact was also shown by use of a loud-speaking telephone. The frequency of the flashes can be varied by altering the capacity and resistance of the circuit. In a particular case a resistance of 1 megohm and a capacity of 1 mf. on a 200-volt supply gave a frequency of about 1·2 flashes per second.

The phenomenon may be explained as follows: When cold the lamp fails to pass any current until subjected to an E.M.F. of about 171 volts. If, however, the voltage be reduced when the lamp is glowing, current continues to pass until about 140 volts is reached, when the current falls suddenly to zero. When, therefore, an E.M.F. of 200 volts is applied, some time elapses while the condenser is charging up to the necessary 171 volts. At the end of this time the lamp begins to take current, the current increases and the voltage across the lamp falls to the limit of 140 volts, when the lamp goes out. This cycle of operations then repeats indefinitely.

It was suggested that the arrangement might be used at low frequency for flashing signs for advertising, &c., at audio-frequency for telephonic measurements, and at high frequency (up to about 15,000 ~) for radio-signalling.

The phenomenon was discovered by accident by Mr. Anson.

DISCUSSION.

DR. E. H. RAYNER: It will be most valuable to have an account of this interesting phenomenon in the records of the Society. I should imagine that the highest frequency as yet obtained with the apparatus—viz., 15,000 ~—is by no means an upper limit, as I have seen a neon lamp taking 60,000 ~. A tube 8 inches long was rotated about a perpendicular axis through its middle point at a distance of 1 foot from a C.W. aerial. The apparent illumination was thus drawn out into a disk with radial striæ, the spacing of which indicated the frequency I have mentioned.

DR. J. H. VINCENT: Can the author tell us whether the lamp exhibits any degree of unilateral conductivity? If so, a good many possible applications suggest themselves. It would also be interesting to know how far the critical voltages depend on the construction of the lamp. The usefulness of the arrangement would be greatly extended if the working voltage could be brought down to, say, 12 volts by suitable design. Has the author made any experiments with suitably arranged inductances instead of condensers?

Mr. J. W. RYDE: Some results which I have obtained may help to answer the last speaker's questions. It appears that the electrical quantities concerned are related as follows:

$$I=(V-e)/\{(MA)^{-1}+R\}$$

where

I = steady current through lamp,
V = voltage across lamp,
e = constant (viz., the extrapolated intercept on the V-axis of the straight part of the I/V curve when produced),
M = a constant,
A = area of cathode,
R = resistance fitted in series with the lamp.

It will be seen that if the electrodes be of unequal area, the conductivity will be unilateral. The constant M is of the order 10^{-4}, and depends on the shape and material of the electrodes, the

nature and purity of the gas in the lamp, and the pressure. It is particularly affected by occluded impurities. If the gas be pure, there is no change of current with time ; but the current drops to an asymptote with time if impurities be admitted—as they are in practice in order to reduce the critical voltage and to prevent sputtering and consequent blackening of the glass.

Prof. C. L. FORTESCUE : I must congratulate the authors on their interesting exhibit. I should like to know whether the frequency is constant, say, to one part in a thousand or ten thousand. As regards the use of the apparatus for wireless transmission, the question of efficiency arises, and my impression is that there would be considerable waste of energy. As to priority of discovery, the late William Duddell used a somewhat similar arrangement in a demonstration at the Royal Institution about 11 years ago ; but, instead of a lamp such as those before us, he employed a bulb which had been improvised for the purpose.

Mr. F. E. SMITH : We have been told that Mr. Anson discovered this phenomenon " by accident," but such accidents only happen to observant persons. I should say the arrangement would be useful for low rather than high frequencies. For the latter purpose the thermionic valve is perfectly satisfactory, while low frequencies can only be obtained with it by using cumbrous inductances and condensers. If the frequency obtained with the neon lamp turns out to be constant, the apparatus might be used for measuring frequency.

Dr. G. B. BRYAN inquired whether the pressure in the bulb remained constant during the life of the lamp.

Mr. W. C. S. PHILLIPS (communicated) : In the case of an " Osglim " lamp used as a night-light, I have noticed some curious facts which may be of interest. When current is switched on, as much as 10 seconds may pass before the lamp lights up. The supply is A.C., but when the frequency of the illumination is tested by moving the hand rapidly, one gets the impression that this latter frequency is somewhat lower than that of the supply. Again, when current is off, so that one electrode is alive and the other insulated, a glow can be noticed on the glass of the bulb in any region to which the hand is approached, as if a capacity-current were flowing through the body of the observer.

The PRESIDENT, remarking that Mr. Anson had begun making discoveries very early in life, invited the authors to undertake a written account for the Society's Proceedings.

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